Coded random access: Using coding theory to build random access protocols

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Enrico Paolini
Dept. of Electrical, Electronic, and Information Engineering
“Guglielmo Marconi”
University of Bologna, Italy

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Outline

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Random Access?

- Next generation wireless networks:
  - Very large number of users
  - Sporadic and unpredicatble user activity
  - Small amount of data per user
- Examples:
  - Massive M2M
  - IoT
- Coordinated access is difficult and/or very inefficient
- Renewed interest for random multiple access schemes

Outline

1. Introduction
2. Background
3. A bridge with coding
4. Results

Some Random Access Schemes

- ALOHA [A70]: Initial unslotted version
- Slotted ALOHA (SA) [R75]: Slotted version
- Framed ALOHA (FA) [OIN77]: Framed version
- Diversity slotted ALOHA (DSA) [CR83]: Twin replicas
- Contention resolution diversity slotted ALOHA (CRDSA) [CGH07]: Twin replicas and successive interference cancellation (SIC)

[A70] N. Abramson, "The ALOHA system - another alternative for computer communications," in *Proc. of 1970 Fall Joint Computer Conf.*, vol. 37, pp. 281–285, AFIPS Press, 1970

[R75] L. G. Roberts, "ALOHA packet system with and without slots and capture," SIGCOMM Comput. Commun. Rev., vol. 5, pp. 28Ð42, Apr. 1975

[OIN77] H. Okada, Y. Igarashi, and Y. Nakanishi, "Analysis and application of framed ALOHA channel in satellite packet switching networks - FADRA method," Electron. Commun. in Japan, vol. 60, pp. 60Ð72, Aug. 1977

[CR83] G. Choudhury and S. Rappaport, "Diversity ALOHA - a random access scheme for satellite communications," *IEEE Trans. Commun.*, vol. 31, pp. 450–457, Mar. 1983

[CGH07] E. Casini, R. De Gaudenzi, and O. del Rio Herrero, "Contention resolution diversity slotted ALOHA (CRDSA) An enhanced random access scheme for satellite access packet networks.," *IEEE Trans.Wireless Commun.*, vol. 6, pp. 1408–1419, Apr. 2007

User Activity Model

Average load

- Framed and slotted random access scheme, M slots per frame – Users are frame- and slot-synchronous
- Population of users of cardinality N (usually large).
- At the beginning of a frame each user is active with activation probability π (constant, usually small)
- Users become active independently of each other; each active user has a packet to transmit in the frame
- Number of active users N_a, s.t. $\mathbb{E}[N_a] = \pi N$
- Average load:

$$\mathsf{G} = \frac{\pi N}{M}$$

CRDSA

Transmission and reception

- Each active user:
  - Generates a replica of his packet
  - Picks two slots in the frame randomly (no coordination among users)
  - Equips each replica with the index of the slot where the copy is sent
  - Transmits the twin packets
- The receiver:
  - If it detects and decodes a packet, it cancels the interference contribution caused by the twin packet on the indexed slot
  - Iterates this procedure, hopefully yielding the recovery of the whole set of packets

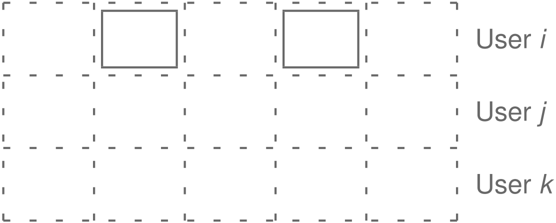
CRDSA

- Example

- Storage capability
- Signal processing

CRDSA

- Example



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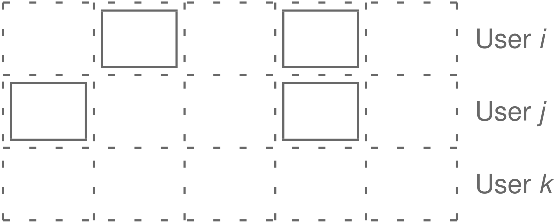

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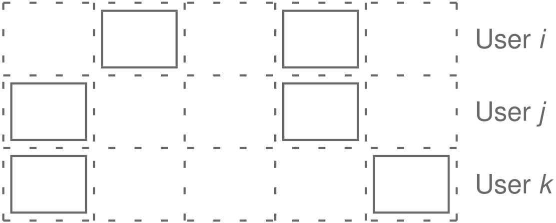

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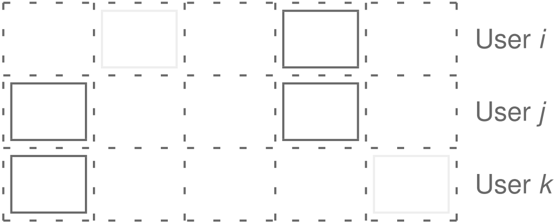

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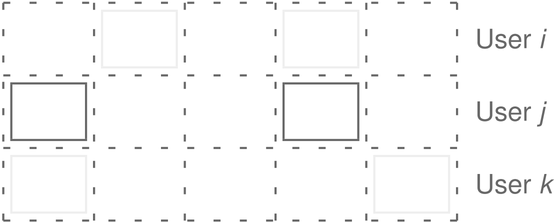

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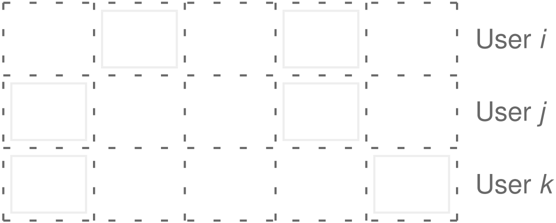

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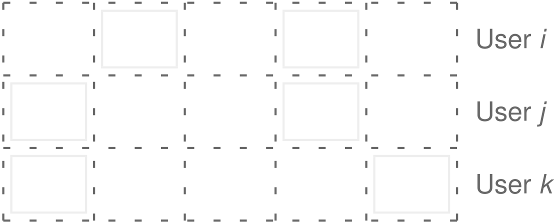

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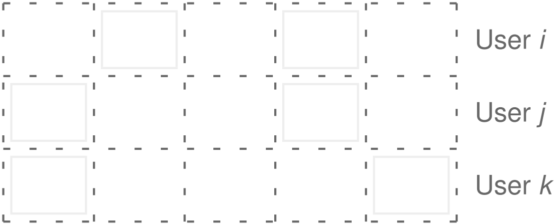

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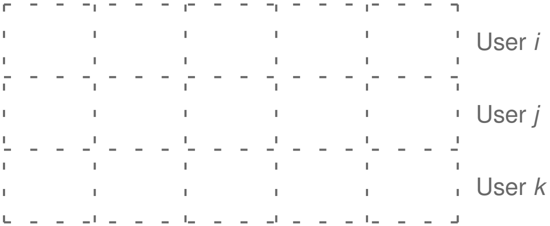
Outline

Irregular Repetition Slotted ALHOA (IRSA)

- Lets users employ variable repetition rates [L11]



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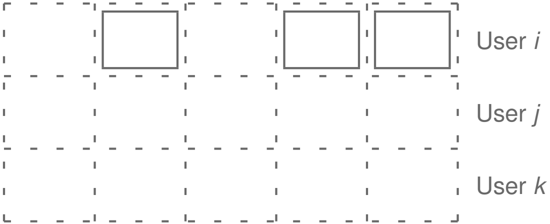

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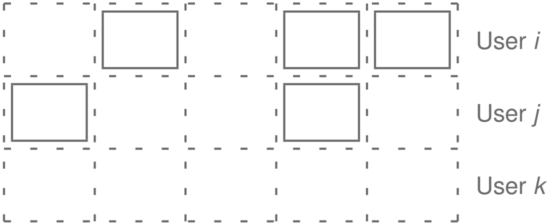

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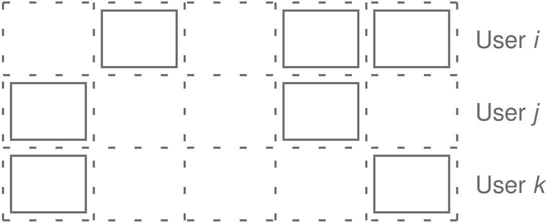

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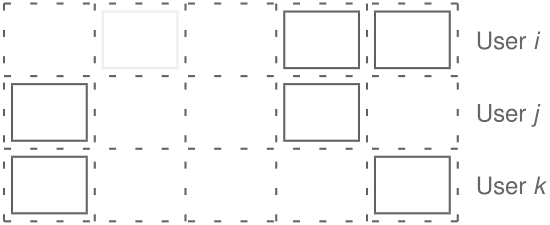

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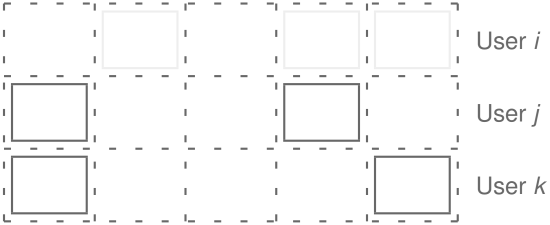

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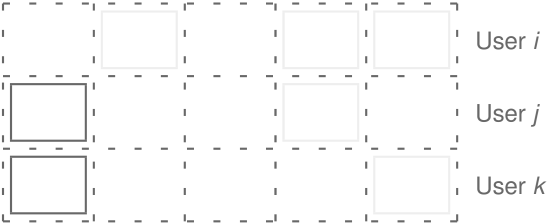

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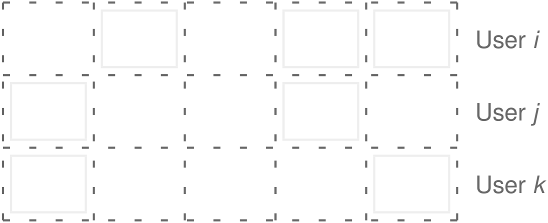

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IRSA

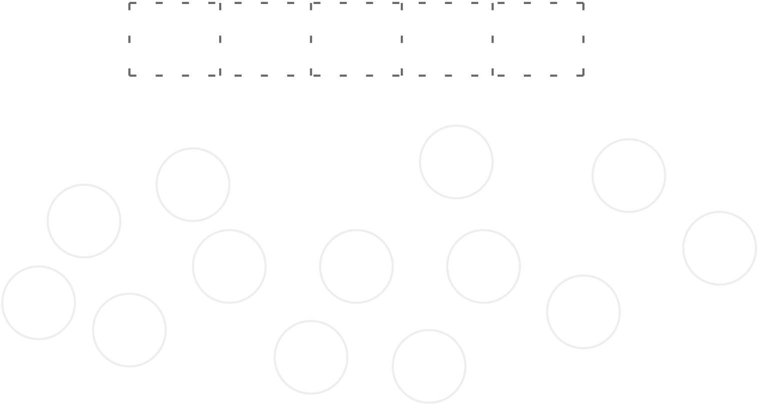
Transmission procedure

- Defined by:
  - A set $\mathcal{D} = \{2, 3, \ldots, d_{\max}\}$ of repetition rates, known to the receiver
  - A p.m.f. $\Lambda = \{\Lambda_2, \Lambda_3, \ldots, \Lambda_{d_{\max}}\}$ on $\mathcal{D}$
- Each active user draws a repetition rate $d \in \mathcal{D}$ according to the p.m.f. Λ
- The user then transmits *d* packet replicas in *d* slots chosen randomly
- Active users choose their repetition rates without any coordination with the other active users
- IRSA rate

$$\mathsf{R} = \frac{1}{\sum_d \Lambda_d \, d} \leq 1/2 \, .$$

IRSA

A graph perspective



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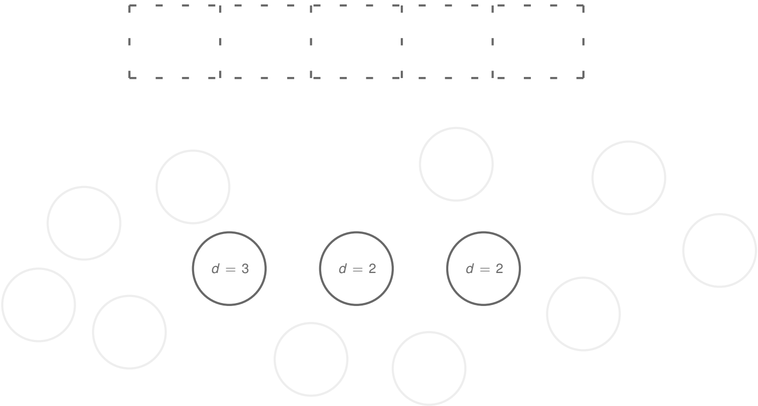

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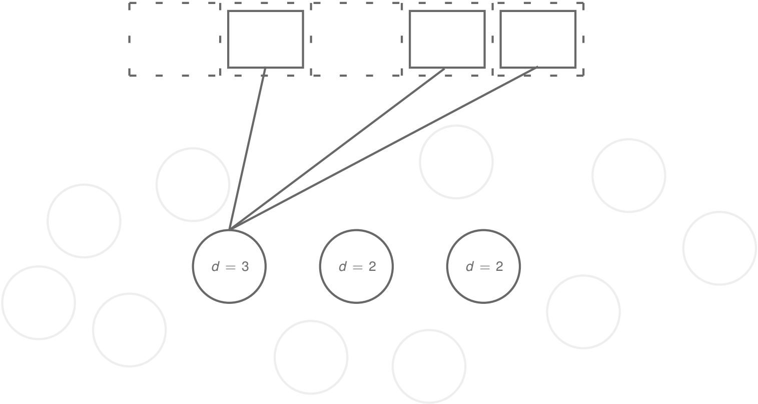

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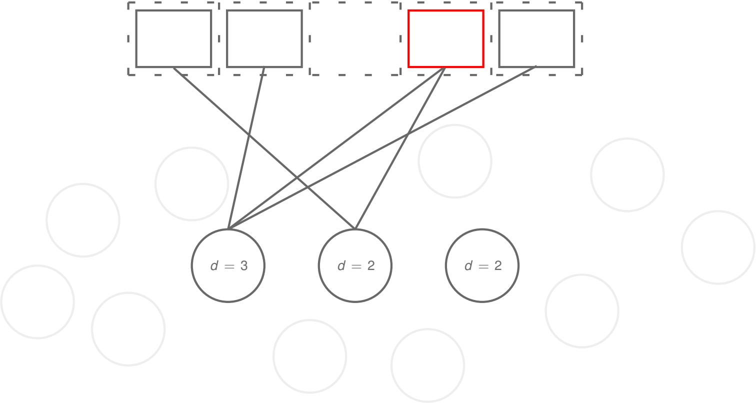

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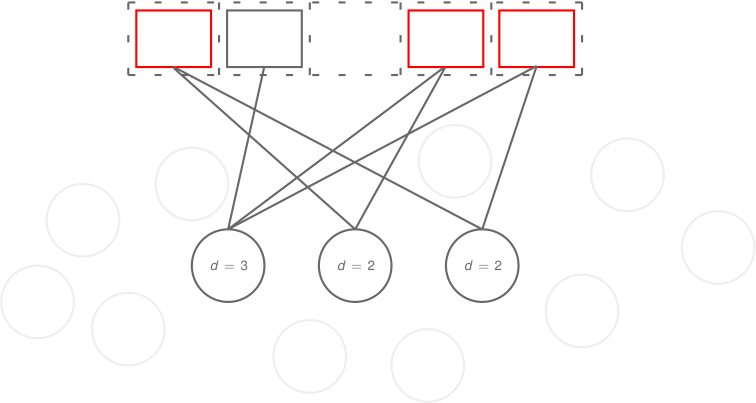

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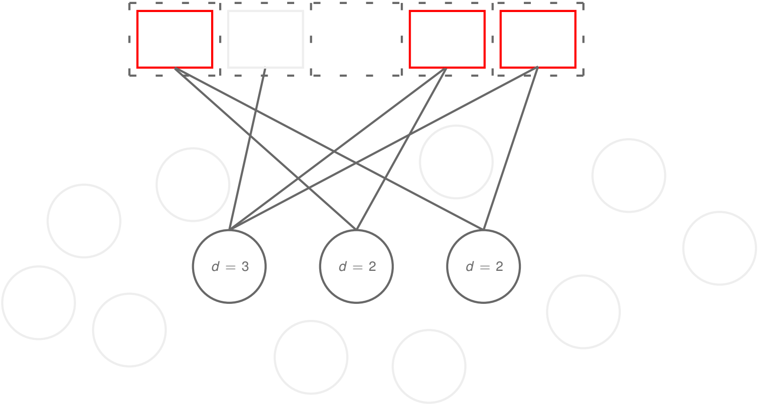

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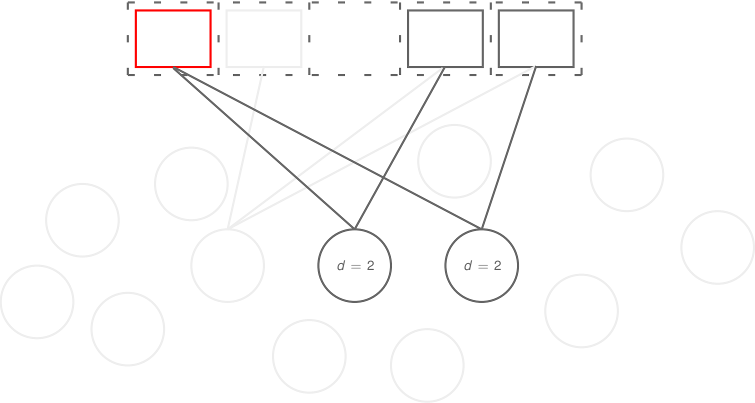

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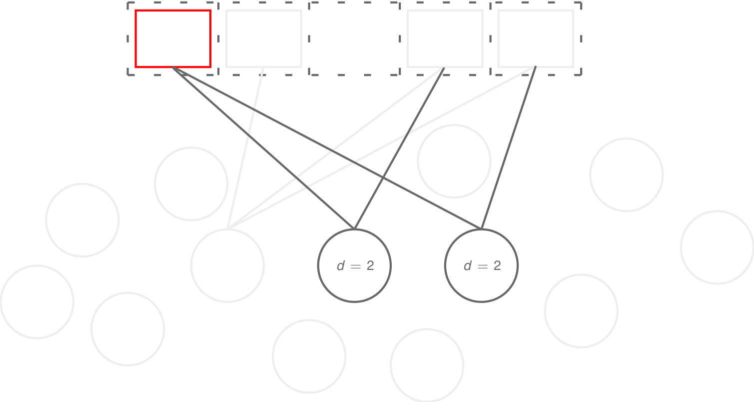

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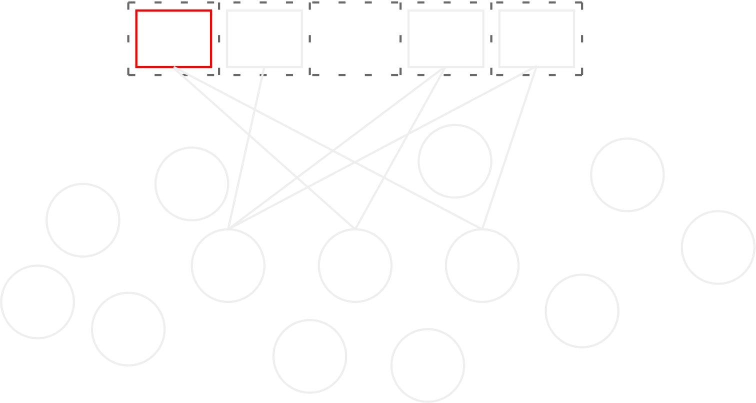

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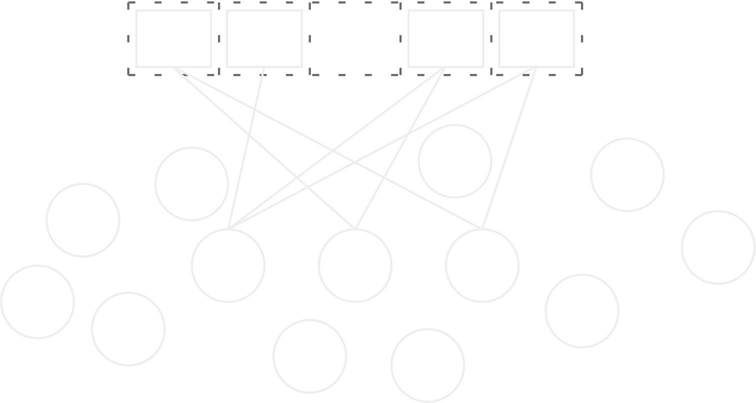

“Collision” Channel

- In each slot the decoder is able to discriminate between:
  - an idle
  - a singleton
  - a collision (no information about collided packets)
- Singleton packets are correctly received
- Interference cancelation is ideal, as so is the estimation of the channel parameters necessary to perform it

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- Over a collision channel IRSA SIC equivalent to erasure decoding of low-density generator matrix (LDGM) or Luby-Transform (LT) codes
  - Analysis
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Coded Slotted ALOHA (CSA)

Transmission procedure

- Generalization of IRSA and CRDSA
  - A set $\mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_{n_c}\}$ of n_c linear block component codes (same dimension k), known to the receiver
  - A p.m.f. $\Lambda = \{\Lambda_1, \Lambda_2, \dots, \Lambda_{n_c}\}$ on $\mathcal{C}$
  - Frames composed of kM sub-slots
- Each active user, independently of the others:
  - Splits his packet into k sub-packets
  - Draws $\mathcal{C}_h(n_h, k) \in \mathcal{C}$ according to Λ
  - Generates n_h encoded sub-packets via $\mathcal{C}_h$
  - Equips sub-packets with the appropriate information
  - Transmits in n_h randomly chosen sub-slots
- CSA rate:

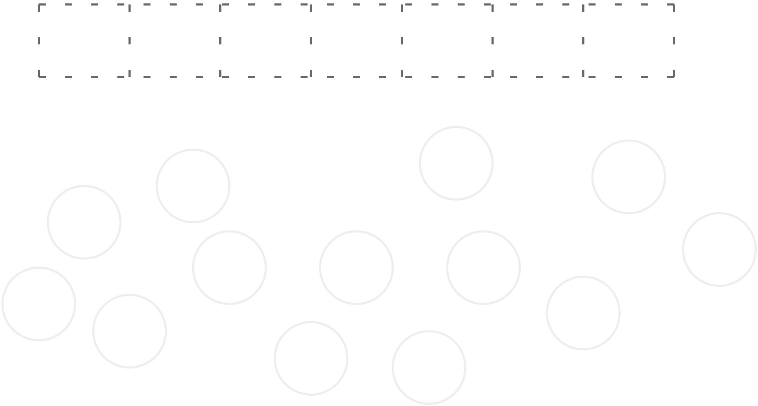
$$\mathrm{R} = \frac{k}{\sum_h \Lambda_h n_h} = \frac{k}{\bar{n}}, \qquad 0 < R \leq \frac{k}{k+1}$$

[PLC15] E. Paolini, G. Liva, M. Chiani "Coded slotted ALOHA: A graph-based method for uncoordinated multiple access" *IEEE Trans. Inf. Theory*, vol. 61, no. 12, 2015

CSA

Example

- Similar to decoding of doubly generalized LDPC codes [PFC10]



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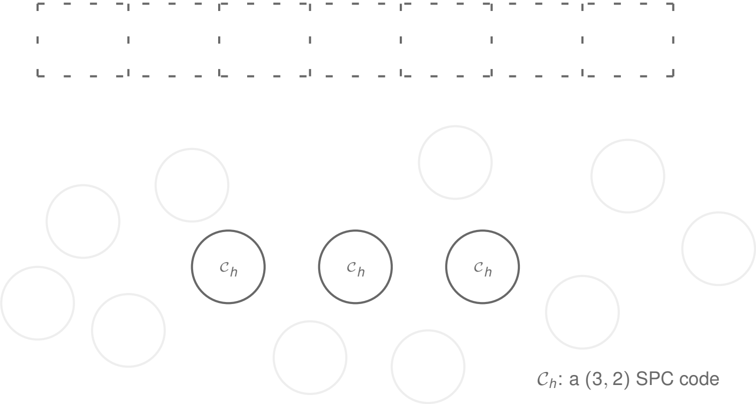

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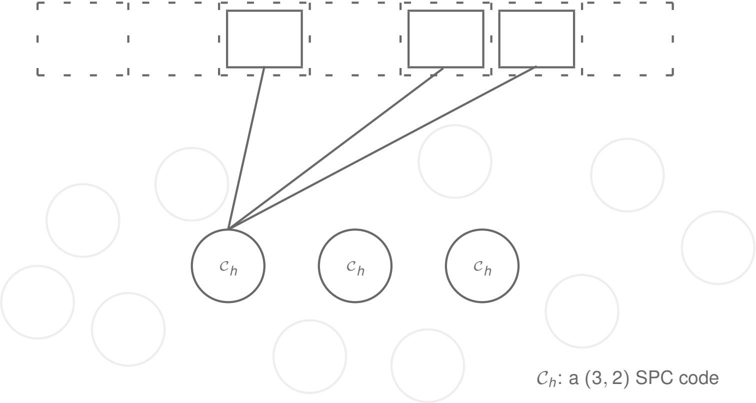

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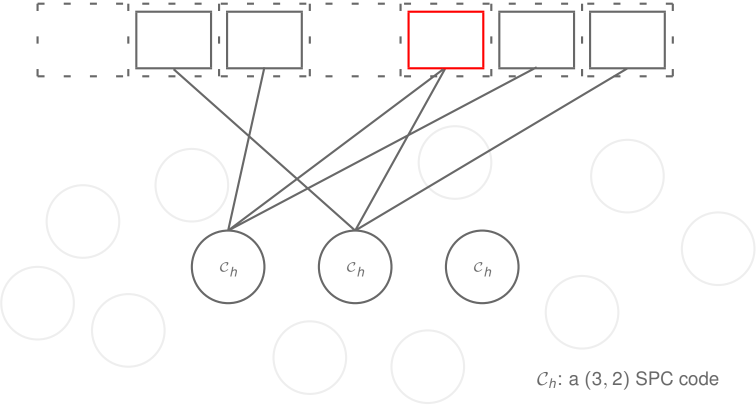

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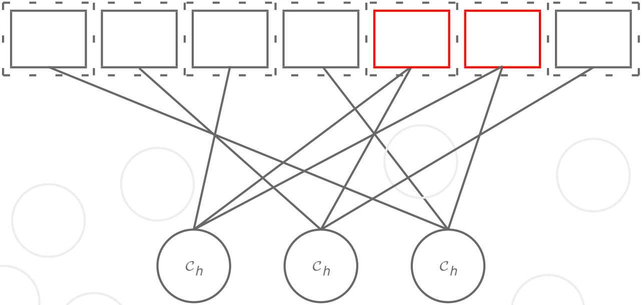

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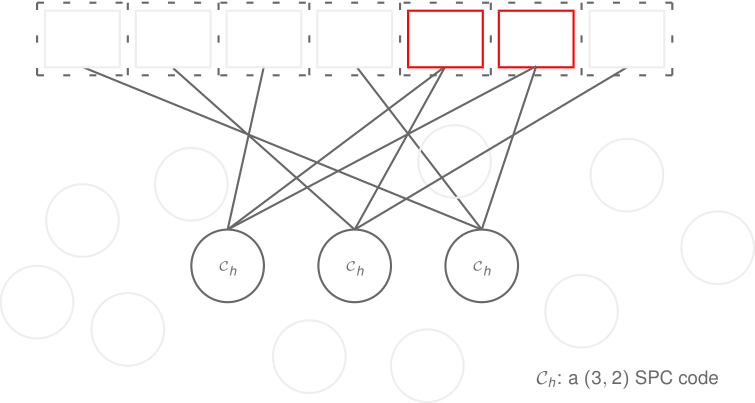

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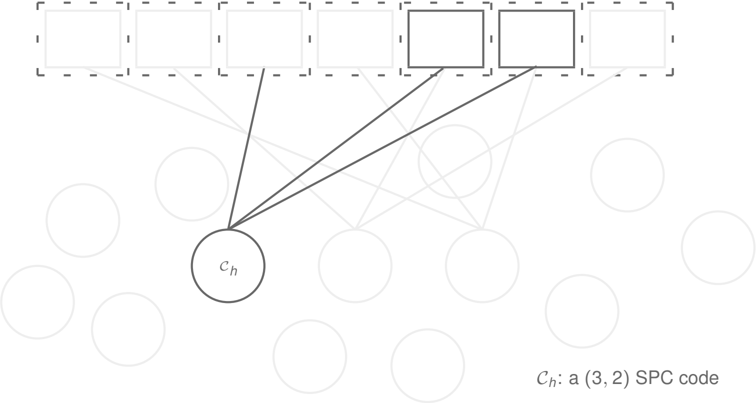

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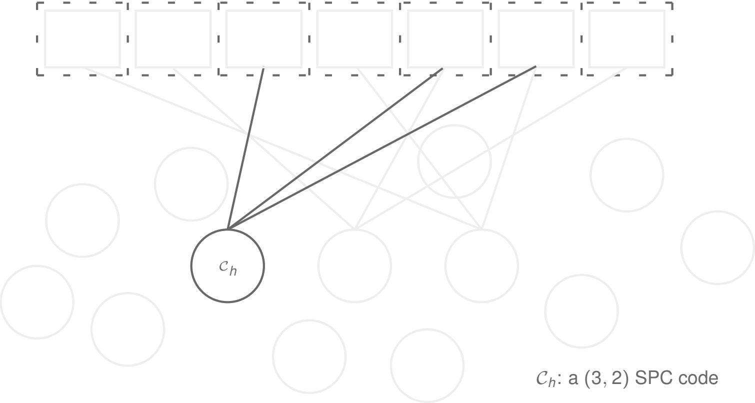

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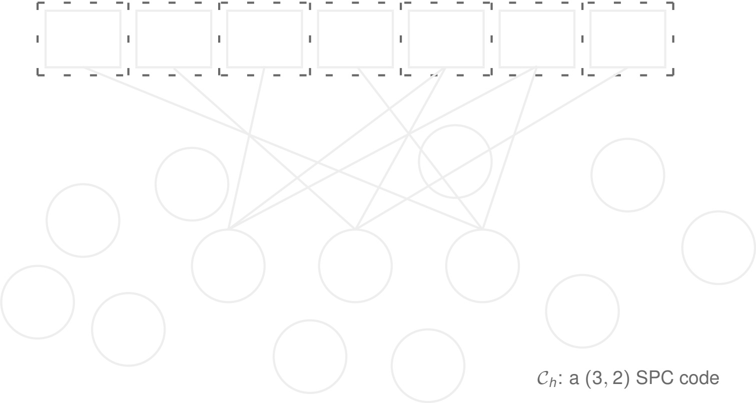

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CSA
Analysis

- Let $N \to \infty$ and $M \to \infty$ for constant π and $\mathsf{G} = \pi N/M$
- MAP decoding used for each component code
- For the generic edge e (ℓ the SIC iteration index):

$$p_\ell = \Pr\{e \text{ connected to a (sub-)slot where a collision persists}\}$$

$$q_\ell = \Pr\left\{\begin{array}{l} e \text{ connected to a user whose contribution of interference} \\ \text{on the corresponding (sub-)slot cannot yet be cancelled} \\ \text{after MAP component decoding} \end{array}\right\}$$

- Then:

$$q_\ell = \frac{1}{\bar{n}} \sum_{h=1}^{n_c} \Lambda_h \sum_{t=0}^{n_h-1} p_{\ell-1}^t (1-p_{\ell-1})^{n_h-1-t} [(n_h-t)\tilde{e}^{(h)}_{n_h-t} - (t+1)\tilde{e}^{(h)}_{n_h-1-t}]$$
$$=: \mathsf{f}_\mathsf{b}(p_{\ell-1})$$

$$p_\ell = 1 - \exp\left(-\frac{\mathsf{G}}{\mathsf{R}} q_\ell\right) =: \mathsf{f}_\mathsf{s}(q_\ell)$$

CSA

Analysis

- $\tilde{e}_g^{(h)}$: the g-th un-normalized information function of code $\mathcal{C}_h$
- The sum of the ranks of all $k \times g$ submatrices of a generator matrix of $\mathcal{C}_h$ [HKL97] [AKtB04]
- Density evolution recursion:

$$p_\ell = (\mathsf{f}_\mathsf{s} \circ \mathsf{f}_\mathsf{b})(p_{\ell-1})$$
$$p_0 = \mathsf{f}_\mathsf{s}(1)$$

[HKL97] T. Helleseth, T. Kløve, and V. I. Levenshtein, "On the information function of an error-correcting code," *IEEE Trans. Inf. Theory*, Mar. 1997

[AKtB04] A. Ashikhmin, G. Kramer, and S. ten Brink, "Extrinsic Information Transfer Functions: Model and Erasure Channel Properties," *IEEE Trans. Inf. Theory*, Vol. 50, Nov. 2004

CSA

Load threshold

- Load threshold:

$$G^*(\mathcal{C},\Lambda) = \sup\{G \text{ s.t. } p_\ell \to 0 \text{ as } \ell \to \infty\}$$

- $0 < G < G^*(\mathcal{C},\Lambda)$: the residual packet erasure probability tends to zero as the number of IC iterations tends to infinity
- $G > G^*(\mathcal{C},\Lambda)$: SIC fails with a probability approaching 1
- For given $\mathcal{C}$ and given R we can optimize $G^*(\mathcal{C},\Lambda)$ with respect to the p.m.f. Λ:

$$\boxed{\begin{array}{l} \text{maximize } G^*(\mathcal{C},\Lambda) \\ \\ \text{subject to } \mathcal{C} = \{\mathcal{C}_1,\ldots,\mathcal{C}_{n_c}\} \\ \qquad\qquad \dfrac{k}{\sum_{h=1}^{n_c}\Lambda_h n_h} = R \end{array}}$$

- Up to $G(\mathcal{C},\Lambda)$, CSA reliable without retransmissions

CSA

Non-achievable region

- For some $\mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_{n_c}\}$ and $\Lambda = (\Lambda_1, \Lambda_2, \dots, \Lambda_{n_c})$, if $p_\ell \to 0$ as $\ell \to \infty$ then

$$\mathsf{R} \le -\frac{\mathsf{G}}{\log(1-\mathsf{G})}$$

- For some $\mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_{n_c}\}$ and $\Lambda = (\Lambda_1, \Lambda_2, \dots, \Lambda_{n_c})$ yielding a rate R, if $p_\ell \to 0$ as $\ell \to \infty$ then

$$\mathsf{G}^*(\mathcal{C}, \Lambda) \le \mathbb{G}(\mathsf{R})$$

where $\mathbb{G}(\mathsf{R})$ is the unique positive solution to

$$\mathsf{G} = 1 - \exp(-\mathsf{G}/\mathsf{R})$$

in $[0, 1)$

CSA

Non-achievable region

- For some $\mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{n_c}\}$ and $\Lambda = (\Lambda_1, \Lambda_2, \ldots, \Lambda_{n_c})$, if $p_\ell \to 0$ as $\ell \to \infty$ then

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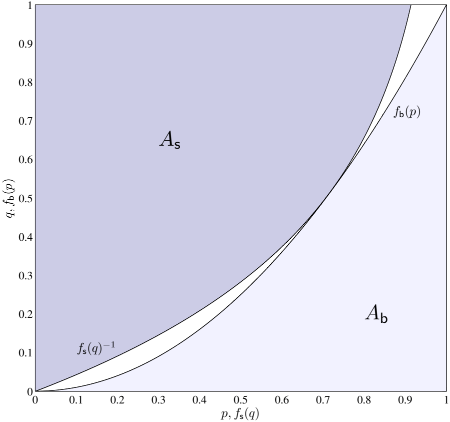
CSA

Non-achievable region



$$A_\mathsf{s} = \int_0^1 f_\mathsf{s}(q)\mathrm{d}q$$

$$= 1 + \frac{\mathsf{R}}{\mathsf{G}} \exp\left(-\frac{\mathsf{G}}{\mathsf{R}}\right) - \frac{\mathsf{R}}{\mathsf{G}}$$

$$A_\mathsf{b} \overset{\mathsf{AT}}{=} \mathsf{R}$$

$$A_\mathsf{b} + A_\mathsf{s} \leq 1$$

AT: by Area Theorem [AKtB04]

Outline

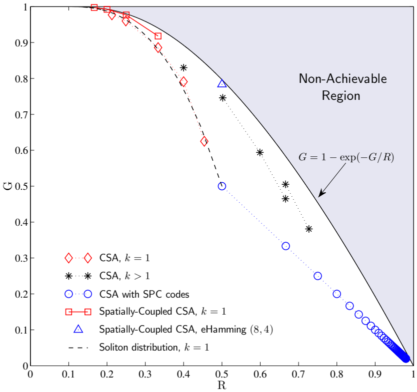
Performance

Load thresholds

	IRSA			
	$R = 1/3$	$R = 2/5$	$R = 1/2$	
$(2, 1)$	0.554016	0.622412	1.000000	
$(3, 1)$	0.261312	0.255176		
$(4, 1)$		0.122412		
$(6, 1)$	0.184672			
$G^*(\mathcal{C}, \Lambda)$	0.8792	0.7825	0.5000	
	CSA $k = 2$, random component codes			
	$R = 1/3$	$R = 2/5$	$R = 1/2$	$R = 3/5$
$(3, 2)$	0.259929	0.304961		0.666667
$(4, 2)$	0.053247	0.144152	1.000000	0.333333
$(5, 2)$	0.447058			
$(6, 2)$		0.347701		
$(7, 2)$		0.203186		
$(11, 2)$	0.105258			
$(12, 2)$	0.134509			
$G^*(\mathcal{C}, \Lambda)$	0.9034	0.8185	0.6556	0.4091
	CSA $k = 3$, random component codes			
	$R = 1/3$	$R = 2/5$	$R = 1/2$	$R = 3/5$
$(4, 3)$	0.173572		0.045538	
$(5, 3)$	0.010699	0.579066		1.000000
$(6, 3)$	0.183304		0.863386	
$(7, 3)$	0.361921		0.091076	
$(8, 3)$	0.025012			
$(10, 3)$		0.025606		
$(11, 3)$		0.395328		
$(18, 3)$	0.245492			
$G^*(\mathcal{C}, \Lambda)$	0.9107	0.8386	0.6868	0.5078
$\mathbb{G}(R)$	0.9405	0.8926	0.7968	0.6758

Performance

Load thresholds

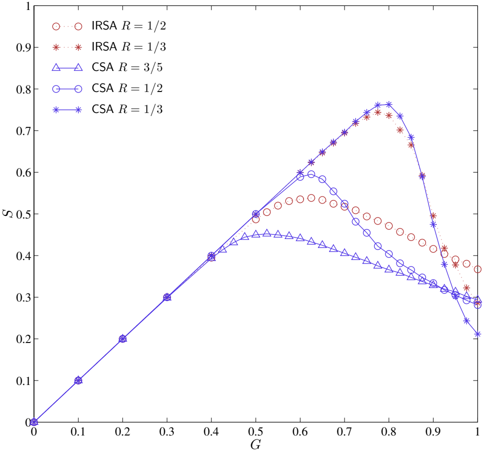


[LPLC12] G. Liva, E. Paolini, M. Lentmaier, and M. Chiani, “Spatially-coupled random access on graphs,” in *Proc. ISIT 2012*

[NP12] K. R. Narayanan and H. D. Pfister, “Iterative collision resolution for slotted ALOHA: An optimal uncoordinated transmission policy,” in Proc. ISTC 2012

Performance

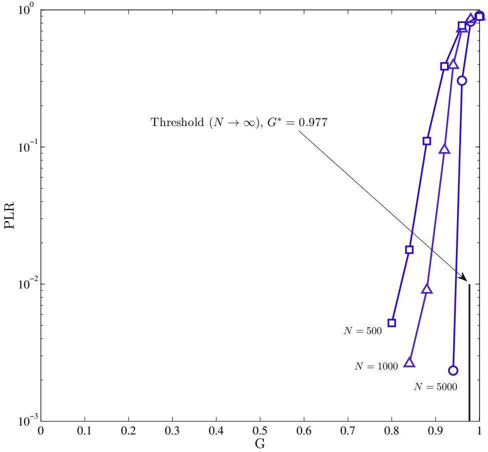
Finite frame length throughput



- $M = 500$ slots
- $N = 20000$ users
- Activation probability $\pi = G\frac{M}{N}$

Performance

Finite frame length packet loss rate



$$\begin{aligned}\Lambda(x) = {} & 0.494155x^2 + 0.159085x^3 + 0.107372x^4 \\ & + 0.070336x^5 + 0.045493x^6 + 0.019898x^7 \\ & + 0.024098x^{11} + 0.008636x^{12} + 0.005940x^{13} \\ & + 0.008749x^{15} + 0.002225x^{18} + 0.001261x^{20} \\ & + 0.002607x^{22} + 0.008092x^{23} + 0.002287x^{24} \\ & + 0.012274x^{25} + 0.002530x^{26} + 0.003094x^{27} \\ & + 0.002558x^{28} + 0.005891x^{29} + 0.013419x^{30}\end{aligned}$$

- IRSA, $M = 500$, 1000, 5000
- 100 iterations

Some Recent (and Less Recent) Results

Coded random access

- E. Paolini, G. Liva, A. Graell i Amat, “A structured irregular repetition slotted ALOHA scheme with low error floors,” in *Proc. ICC 2017*
- F. Clazzer, E. Paolini, I. Mambelli, Čedomir Stefanović, “Irregular repetition slotted ALOHA over the rayleigh block fading channel with capture,” *in Proc. ICC 2017*
- E. Sandgren, A. Graell i Amat, F. Brännström, “On frame asynchronous coded slotted ALOHA: Asymptotic, finite length, and delay analysis,” *IEEE Trans. Commun.*, Feb. 2017
- M. Ivanov, F. Brännström, A. Graell i Amat, P. Popovski, “Broadcast coded slotted ALOHA: A finite frame length analysis,” *IEEE Trans. Commun.*, Feb. 2017
- A. A. Purwita, K. Anwar, “Massive multiway relay networks applying coded random access,” *IEEE Trans. Commun.* Oct. 2016
- A. Taghavi, A. Vem, J.-F. Chamberland, K. Narayanan, “On the design of universal schemes for massive uncoordinated multiple access,” in *Proc. ISIT 2016*
- R. De Gaudenzi, O. Del Rio Herrero, G. Acar, E.G. Barrabés, “Asynchronous contention resolution diversity ALOHA: Making CRDSA truly asynchronous,” *IEEE Trans. Wireless Commun.*, Nov. 2014

Some Recent (and Less Recent) Results

Uncoordinated access

- X. Chen, T.-Y. Chen, D. Guo, “Capacity of Gaussian many-access channel,” *IEEE Trans. Inf. Theory*, to appear
- J. Goseling, Č. Stefanović, P. Popovski, “Sign-compute-resolve for tree splitting random access,” submitted for publication
- S. Madala, K. Narayanan, “Uncoordinated rate selection: Approaching the capacity of Gaussian MAC without coordination,” in *Proc. ICC 2015 Workshops*
- J. Goseling, M. Gatspar, H. Weber, “Random access with physical-layer network coding,” *IEEE Trans. Inf. Theory*, Jul. 2015
- J. Luo and A. Ephremides, “A new approach to random access: Reliable communication and reliable collision detection,” *IEEE Trans. Inf. Theory*, Feb. 2012
- P. Minero, M. Franceschetti, D. Tse, “Random access: An information-theoretic perspective,” *IEEE Trans. Inf. Theory*, Feb. 2012