Title: IT-SP

Sunday, 26 February 2017 12:47

Information-Theoretic Signal Processing

Ram Zamir

Tel-Aviv University
Israel

Madrid IT-School
May 2017

Outline

Wednesday, April 26, 2017 8:56 AM

* Information Theory vrs Signal Processing...

1) Prediction

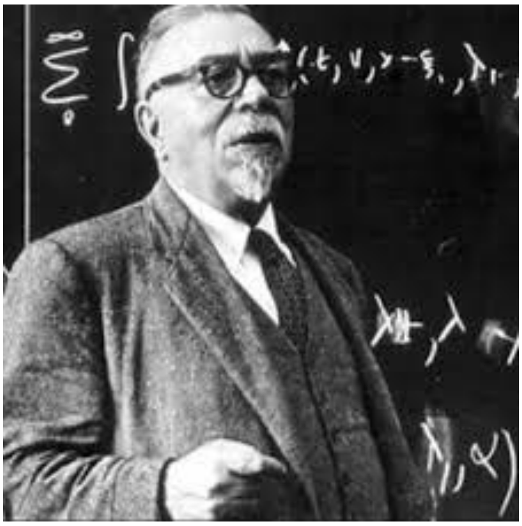
2) Dither & Estimation

3) Oversampling & Noise Shaping

Outline 0

Wednesday, April 26, 2017 8:56 AM

* Shannon meets Wiener !



1) Prediction
2) Dither & Estimation
3) Oversampling & Noise Shaping

Outline 0

Wednesday, April 26, 2017 8:56 AM

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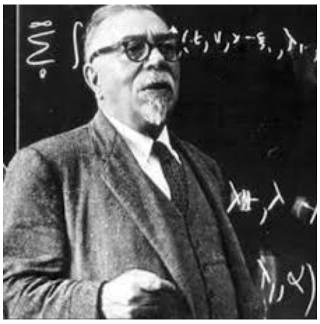

1) Prediction

2) Dither & Estimation

3) Oversampling & Noise Shaping

Outline 1

Wednesday, April 26, 2017 8:56 AM

* Shannon meets Wiener !

1) Prediction

Rate-distortion theory w memory

$\Longleftrightarrow$ Differential Pulse Code Modulation (DPCM)

Channel decoding w memory

$\Longleftrightarrow$ Decision Feedback Equalization (DFE)

2) Dither & Estimation

3) Oversampling & Noise Shaping

Outline 2

Wednesday, April 26, 2017 8:56 AM

* Shannon meets Wiener !?

1) Prediction

Rate-distortion theory w memory $\Longleftrightarrow$ DPCM

Channel decoding w memory $\Longleftrightarrow$ DFE

2) Dither & Estimation

Entropy-Coded Dithered Quantization (ECDQ)

Voronoi Constellation: Lattice coding & decoding

3) Oversampling & Noise Shaping

Outline 3

Wednesday, April 26, 2017 8:56 AM

* Shannon meets Wiener !

1) Prediction

Rate-distortion theory w memory $\Longleftrightarrow$ DPCM

Channel decoding w memory $\Longleftrightarrow$ DFE

2) Dither & Estimation

Entropy-Coded Dithered Quantization (ECDQ)

Voronoi Constellation: Lattice coding & decoding

3) Oversampling & Noise Shaping

Analog-to-Digital conversion (A/D)

Multiple Descriptions

Information Theory - elements

Sunday, 26 February 2017 14:08

* Random Codebook :

$$\{\underline{X}(i)\}_{i=1}^{M}, \quad X(i)_j \sim \text{iid} \sim p(x)$$

* Joint-typicality encoding & decoding

$$\{\underline{X}(i), \underline{y}\} \in A_\varepsilon \triangleq \text{typical set}$$

Elements of Information Theory

Sunday, 26 February 2017 14:08

* Random Codebook :

$\{\underline{X}(i)\}_{i=1}^{M}$, $X(i)_j \sim$ iid $\sim p(x)$

* Joint-typicality encoding & decoding

$\{\underline{X}(i), \underline{y}\} \in A_\varepsilon \triangleq$ typical set

$\Rightarrow$ $n \to \infty$, "0-1 laws"

complicated , less intuitive ...

Elements of Signal Processing

Tuesday, March 07, 2017 9:20 AM

* Linear systems (finite order)

$$y_n = \sum_{k=0}^{K} a_k \cdot x_{n-k} + z_n$$

* Time - frequency

$$X(f) = DFT\{x(t)\}$$

* Estimation, Prediction

$$\hat{x}_n = \sum_{k=1}^{\check{k}} h_k \, y_{n-k} \quad \text{or} \quad \hat{X}(f) = H(f) \cdot Y(f)$$

Elements of Signal Processing

Tuesday, March 07, 2017 9:20 AM

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$\Longrightarrow$ Eeasy to analyze

- any (finite) dimension N & order K !

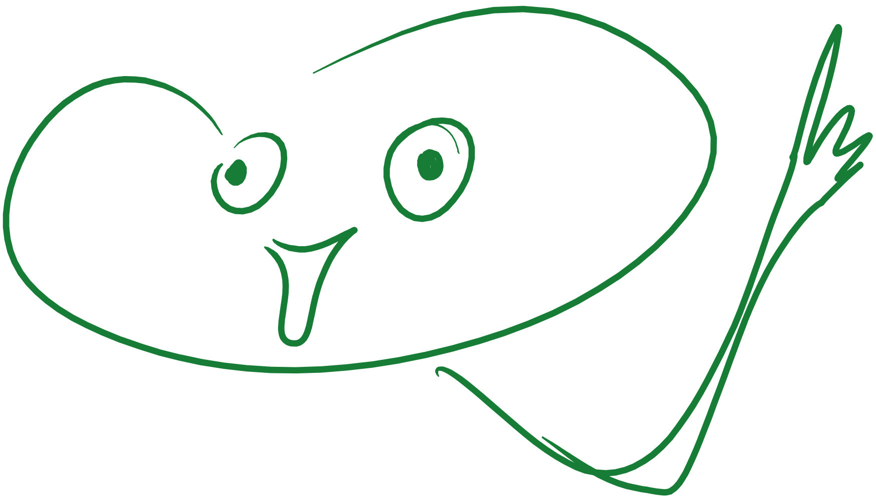
Information-Theoretic Signal Processing

Tuesday, March 07, 2017 12:20 PM

Incorporate SP techniques

into

IT setups



Linear Prediction

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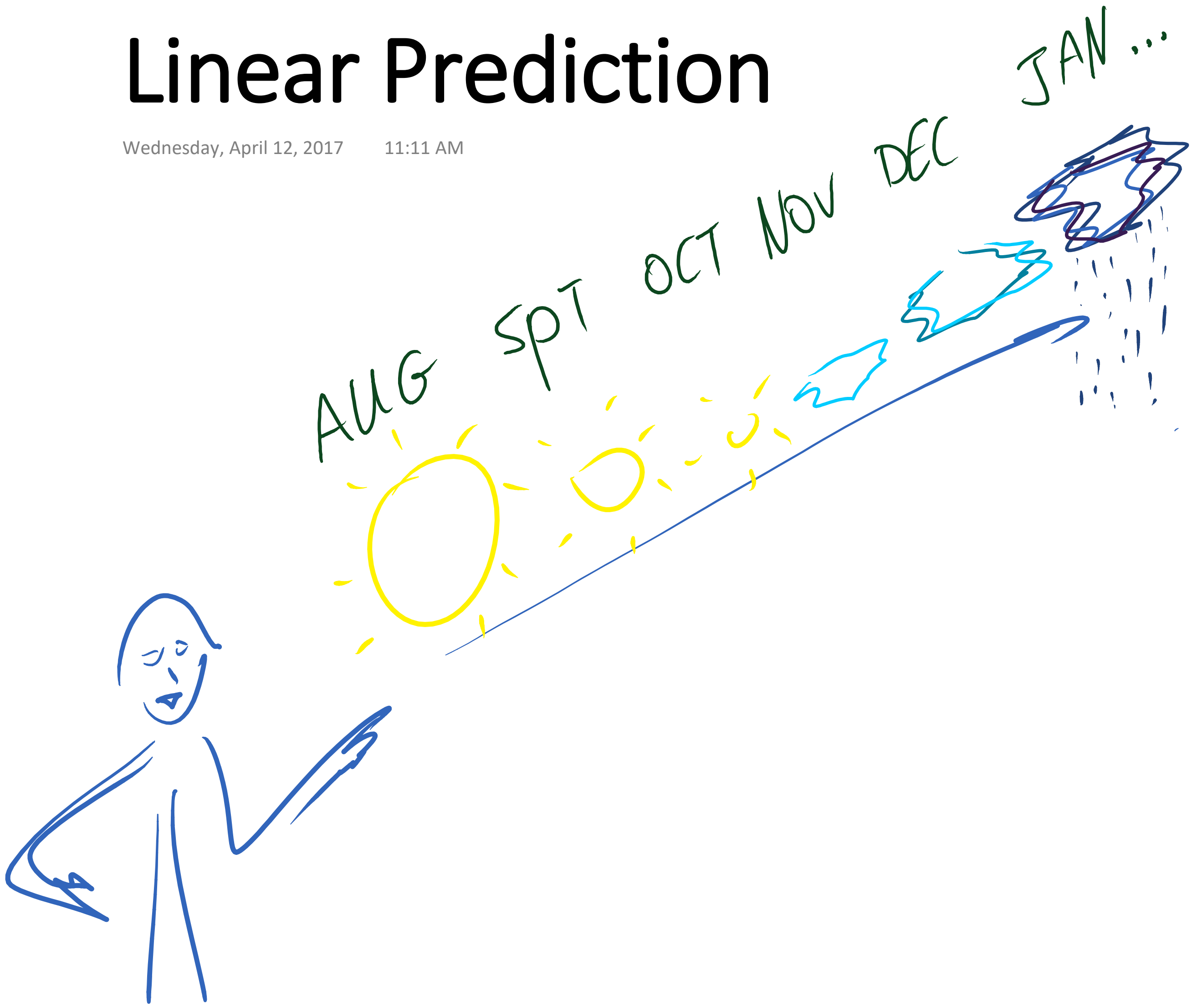


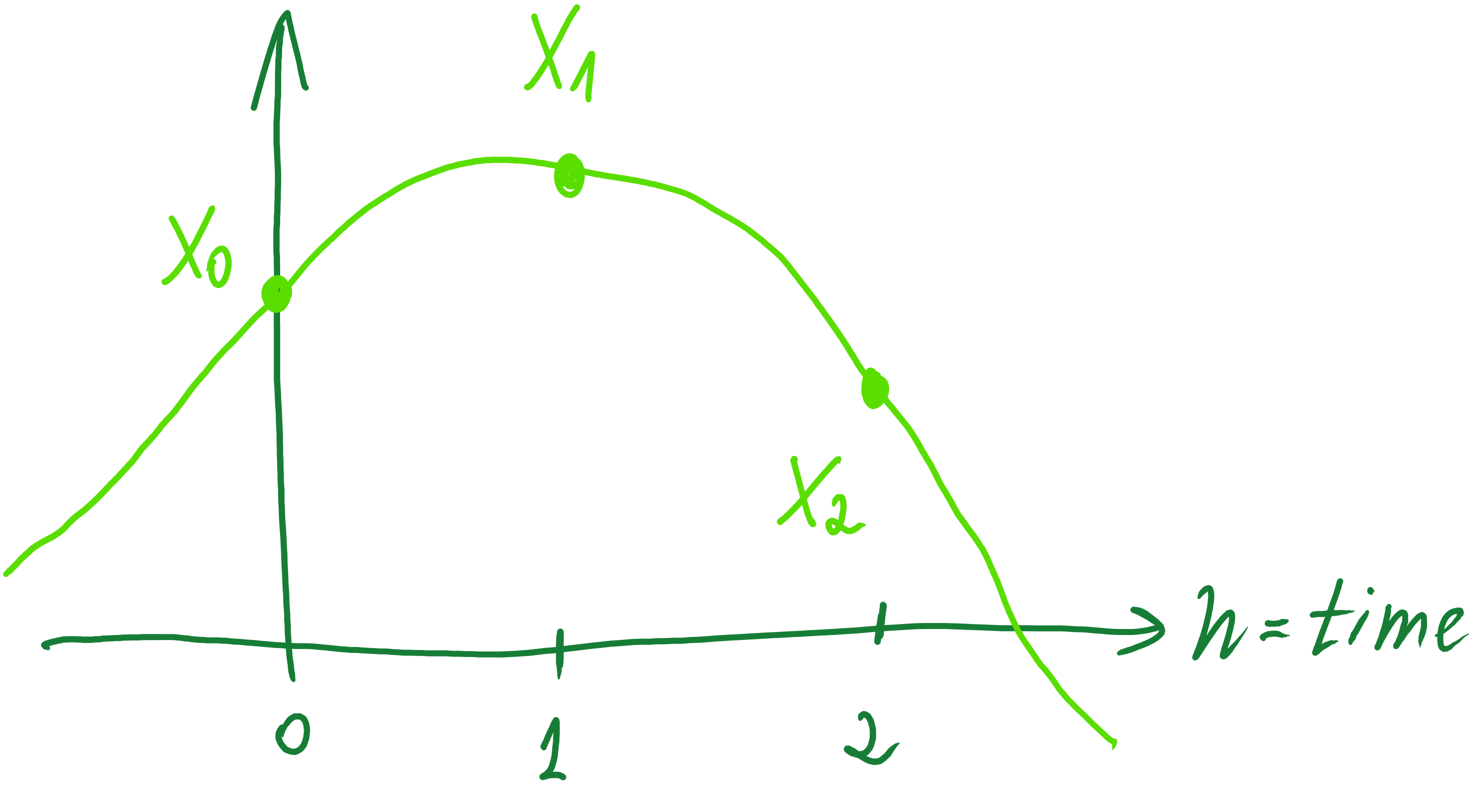
predict new outcome $= X_n$,

given past outcomes $X_{n-1}, X_{n-2}, \dots$

1st-order prediction

Wednesday, April 12, 2017 11:27 AM

$$\hat{X}_n = a \cdot X_{n-1}, \quad n = 1, 2, \ldots$$

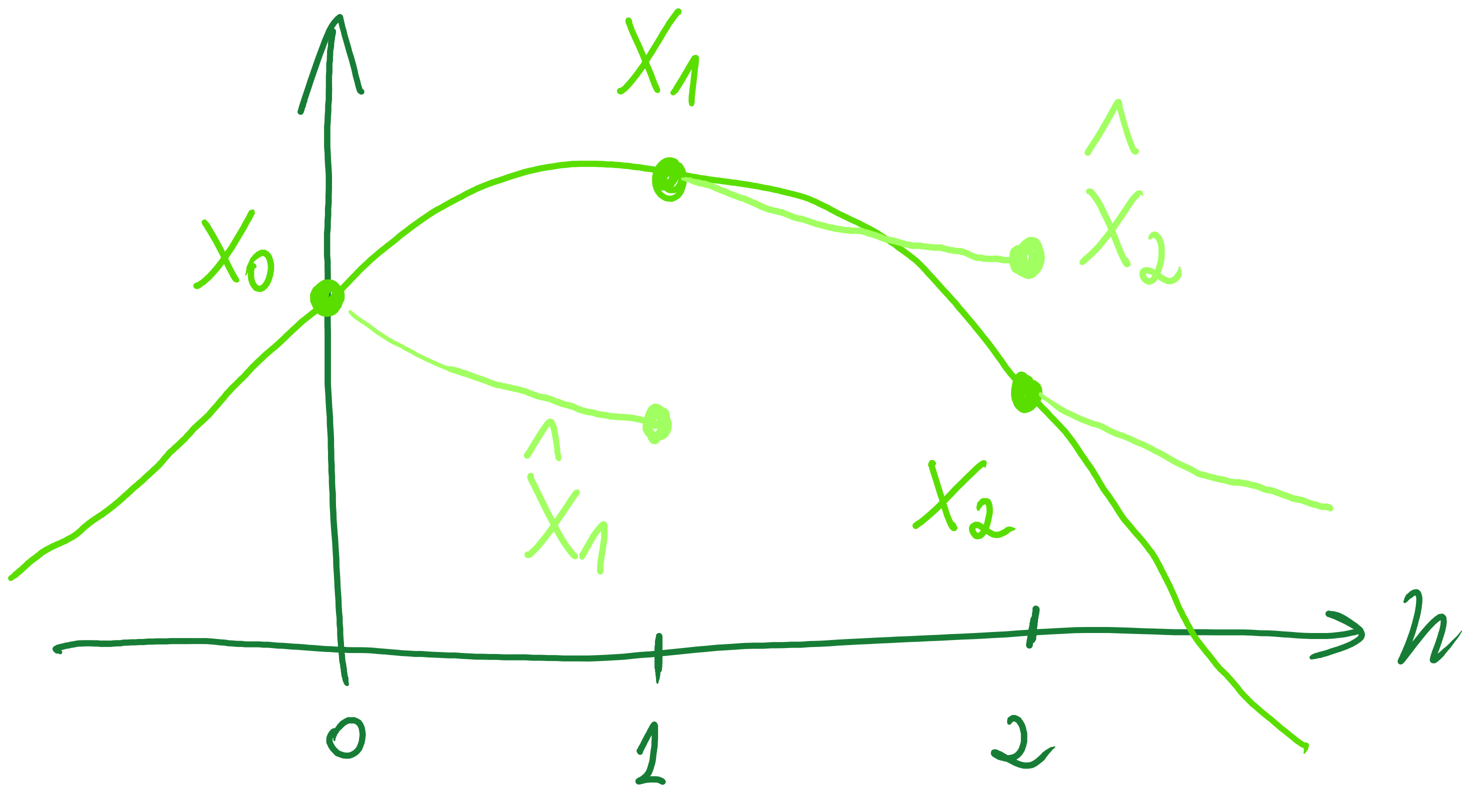


$$X_n = \text{signal}$$

1st-order prediction

Wednesday, April 12, 2017 11:27 AM

$$\hat{x}_n = a \cdot x_{n-1} \quad , \quad n = 1, 2, \ldots$$



x_n = signal

$\hat{x}_n$ = predictions

1st-order prediction

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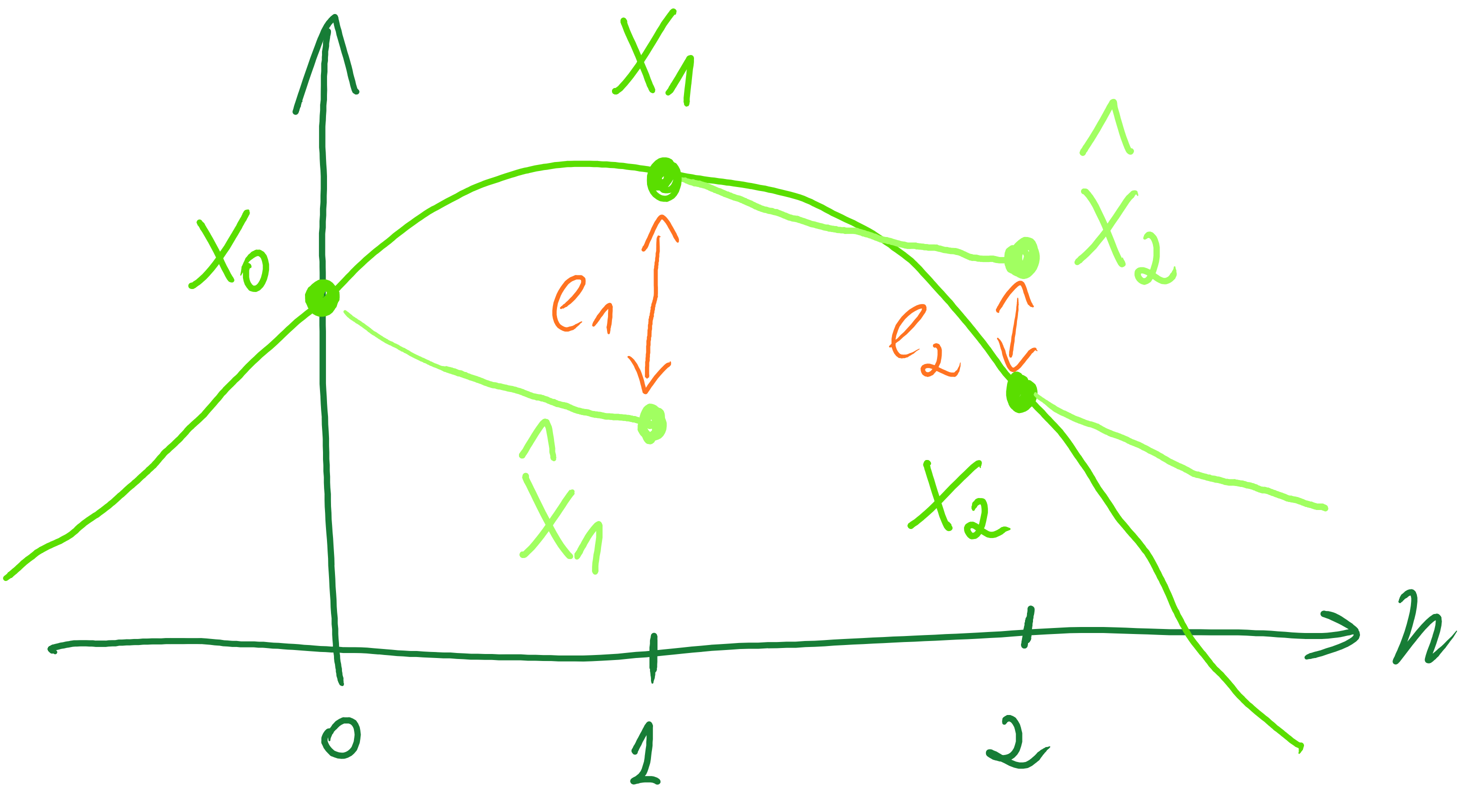
$$\hat{x}_n = a \cdot x_{n-1} \quad , \quad n = 1, 2, \ldots$$



x_n = signal

$\hat{x}_n$ = predictions

e_n = errors

1st-order prediction

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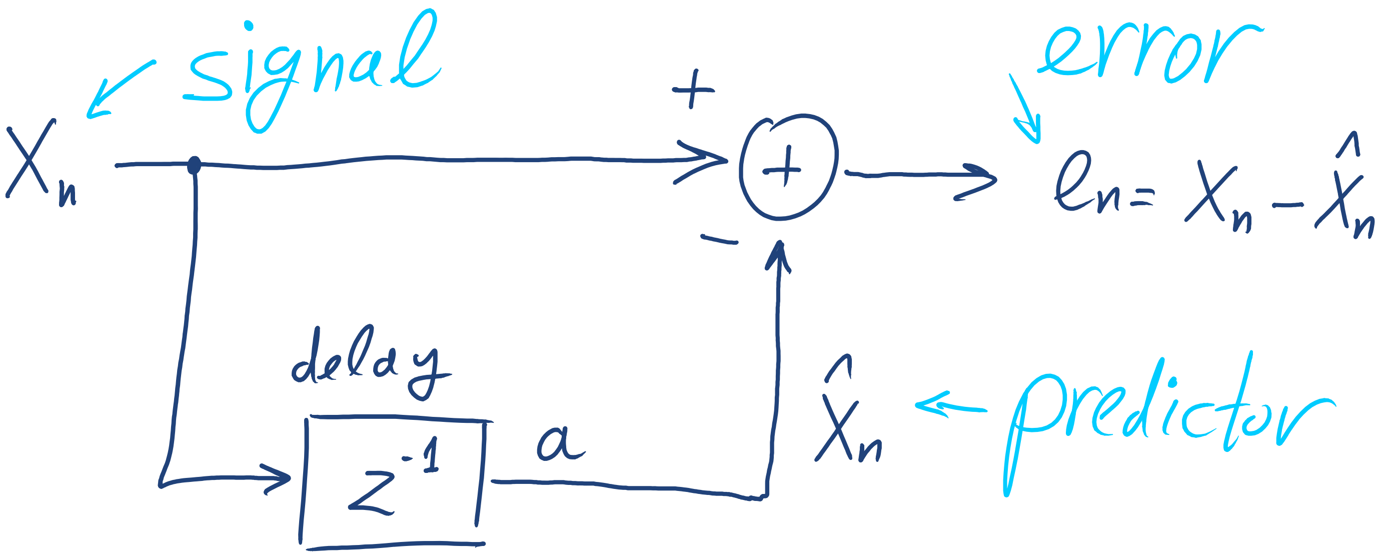
$$\hat{x}_n = a \cdot x_{n-1} , \qquad n = 1, 2, \ldots$$



$$\text{LMMSE}: \quad \min_{a} E\{(\underbrace{x_n - \hat{x}_n}_{e_n})^2\} = ?$$

$$\text{LMMSE} \leq \text{MSE}(@\ a=0) = E\{x_n^2\}$$

Orthogonality Principle

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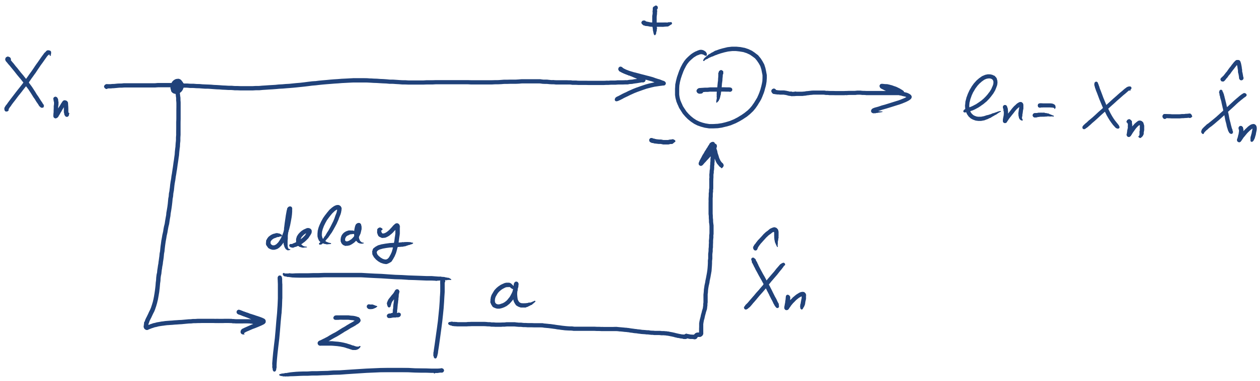
$$\hat{X}_n = a \cdot X_{n-1}, \quad n = 1, 2, \ldots$$



* Orthogonality principle:

If $\hat{X}_n$ is the LMMSE predictor, then

$$\underbrace{e_n}_{\text{error}} \perp \underbrace{X_{n-1}}_{\text{measurement}}$$

Orthogonality Principle (cont)

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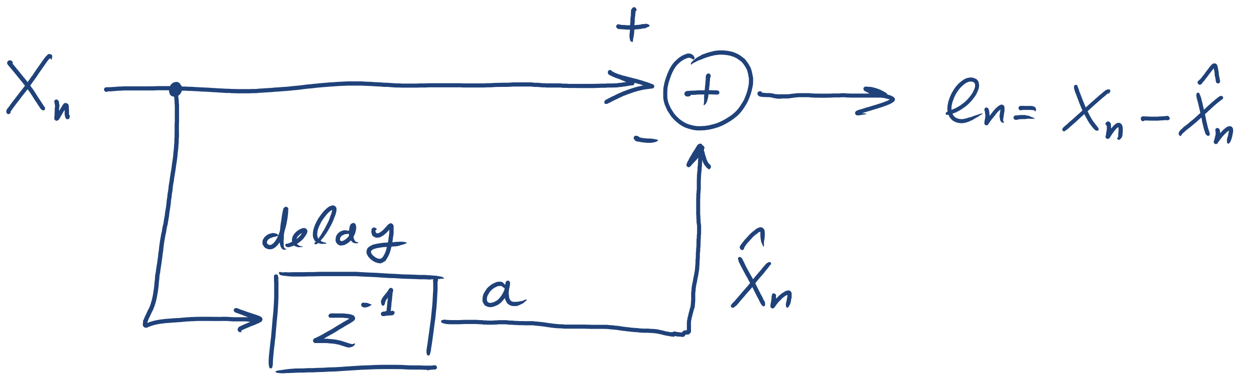
$$\hat{X}_n = a \cdot X_{n-1} \quad , \qquad n = 1, 2, \ldots$$



* Orthogonality Principle:

If $\hat{X}_n$ is the LMMSE predictor, then

$$\underbrace{e_n}_{\text{error}} \perp \underbrace{X_{n-1}}_{\text{measurement}}$$

$$\Rightarrow E\{(X_n - aX_{n-1}) \cdot X_{n-1}\} = 0$$

$$\Rightarrow a^{opt} = \frac{E X_n \cdot X_{n-1}}{E X_{n-1}^2} = ? \ldots$$

Wide Sense Stationary process

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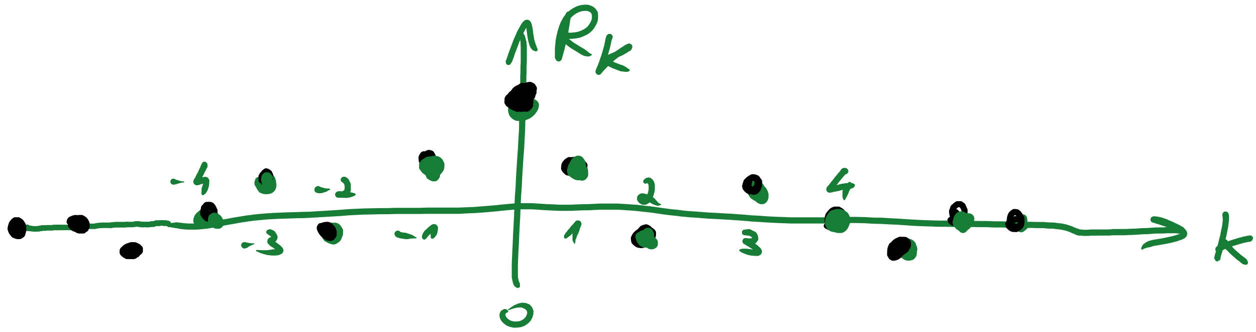
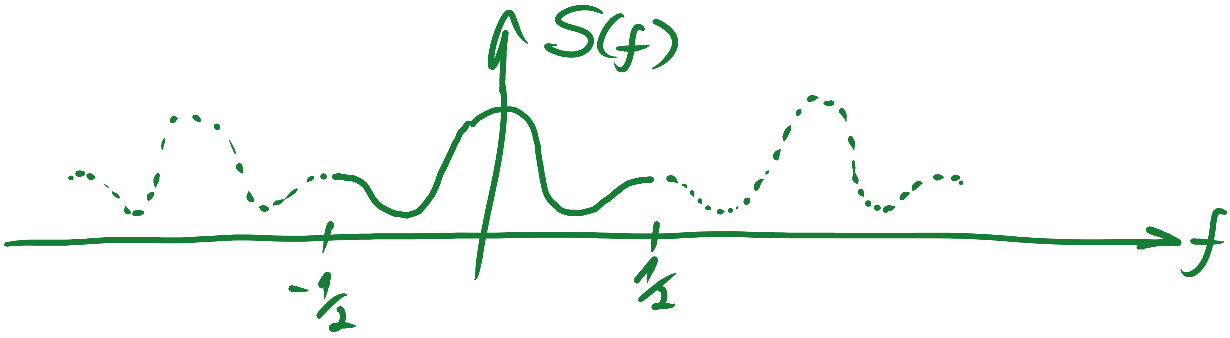
two sided $\ldots X_{-2}, X_{-1}, X_0, X_1, X_2, \ldots$

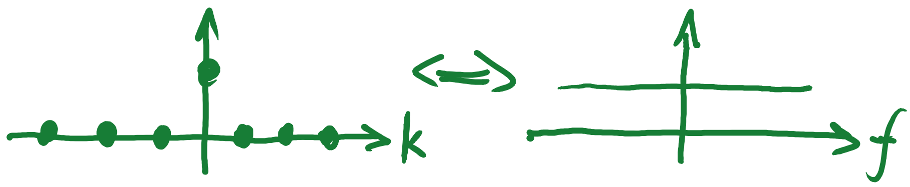
Correlation $R(k) \triangleq E\{X_{n+k} \cdot X_n\}$ invariant of n



Spectrum $S(f) \triangleq \sum_{k=-\infty}^{\infty} R_k \cdot e^{-j2\pi f k}, \quad -\frac{1}{2} \le f \le \frac{1}{2}$



white process $R(k) = 0 \;\; \forall k \neq 0 \Leftrightarrow S(f) = \text{const.} \;\; \forall f$



Prediction of a WSS process

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$$a^{opt} = \frac{E\{X_n \cdot X_{n-1}\}}{E\{X_{n-1}^2\}} = \frac{R(1)}{R(0)} = \rho$$

WSS

ρ = "correlation coefficient" (of X_n & X_{n-1})

$e_n = X_n - \hat{X}_n$

$$\Rightarrow \quad LMMSE = E\{e_n^2\} = E\{e_n \cdot (X_n - \hat{X}_n)\}$$

$$= E\{e_n \cdot X_n\} = R(0) \cdot (1 - \rho^2)$$

orthogo principle

$e_n \perp \hat{X}_n$

$e_n = X_n - \rho \cdot X_{n-1}$

$E X_n X_{n-1} = R(1) = \rho \cdot R(0)$

Pythagorean Relations

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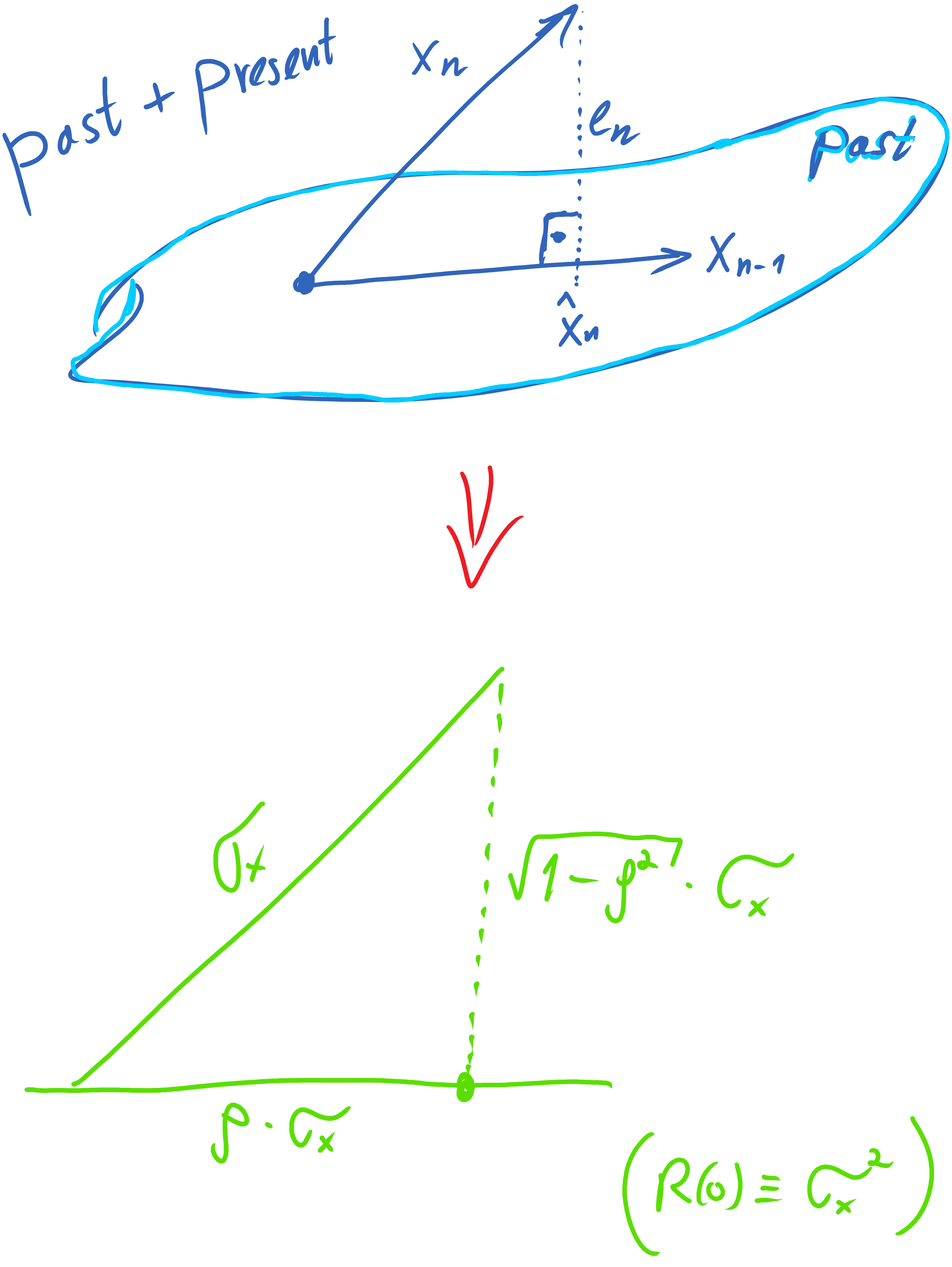

Prediction Gain

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No prediction $(a=0)$ $\Rightarrow$ $e_n \equiv x_n$

$\Rightarrow$ $E\{e_n^2\} = R(0)$

$\therefore$ Prediction gain $\triangleq \dfrac{\text{MSE @ no prediction}}{\text{LMMSE}}$

$$G_1 = \begin{cases} 1, & \text{if white process } (\rho = 0) \\ \frac{1}{1-\rho^2} > 1, & \text{if colored process } (\rho \neq 0) \end{cases}$$

present $\not\perp$ past

present $\perp$ past

General (p-th order) prediction

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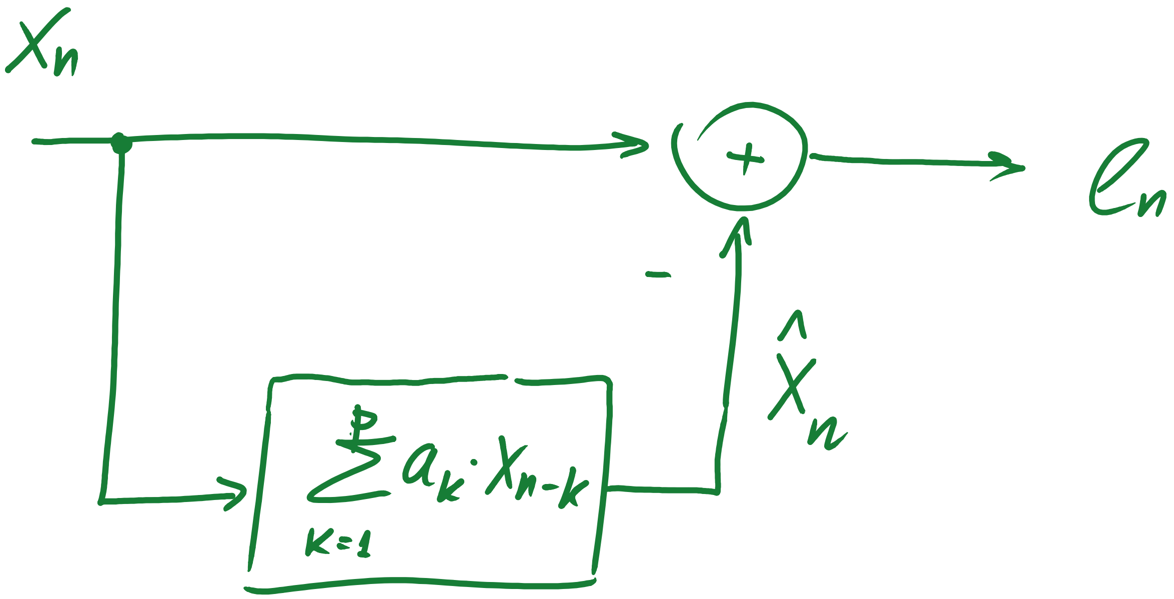

Orthogonality Principle for LMMSE predictor:

$$e_n \perp (x_{n-1}, \ldots, x_{n-p})$$

$$\Rightarrow \quad \underline{a}^{opt} = \underline{\underline{R}}_p^{-1} \cdot \underline{R}_p$$

$$= \text{func}\left\{ \frac{R(1)}{R(0)}, \ldots, \frac{R(p)}{R(0)} \right\}$$

auto correlation & cross correlation

WSS

General (p-th order) prediction

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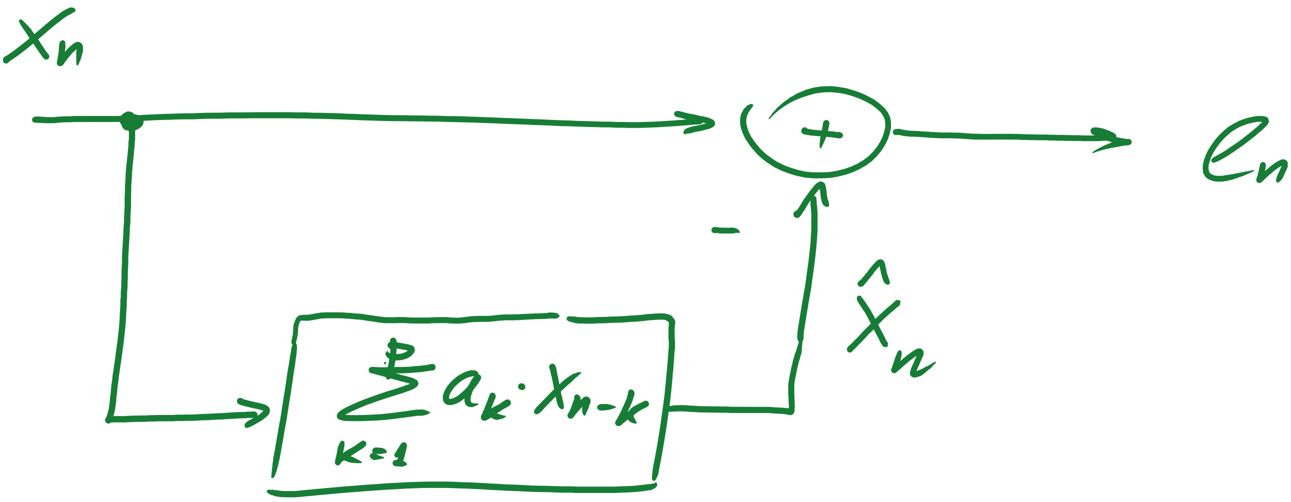
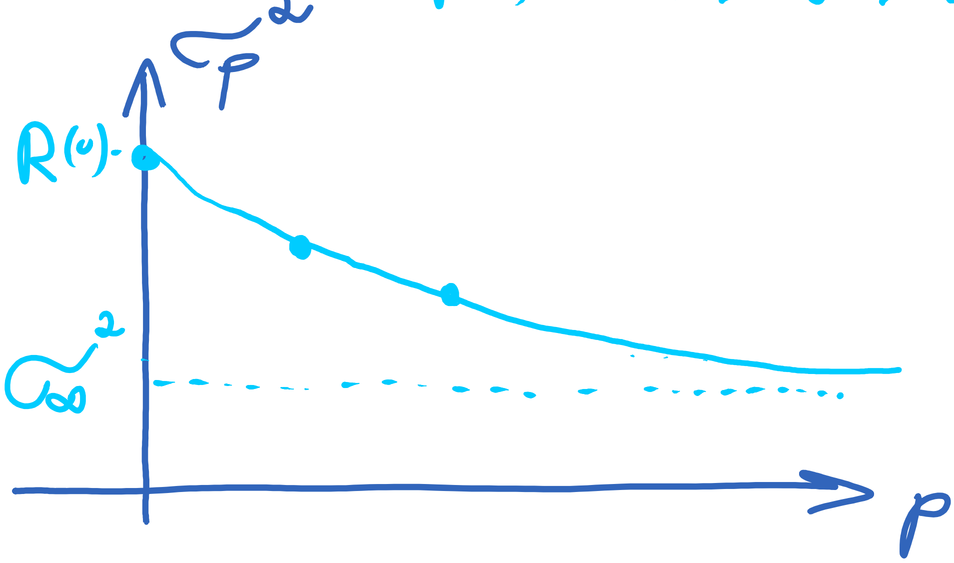


$$\sigma_p^2 \triangleq \text{p-th order prediction MSE} = E\{(e_n)^2\}$$

$$= R(0) - \underline{R}_p^T \cdot \underline{\underline{R}}_p^{-1} \cdot \underline{R}_p$$

$$R(0) \geq \sigma_1^2 \geq \sigma_2^2 \geq \dots \qquad G_p = \frac{R(0)}{\sigma_p^2}$$



$$\sigma_\infty^2 \triangleq \lim_{p\to\infty} \sigma_p^2 = ?$$

Markov (auto-regressive) process

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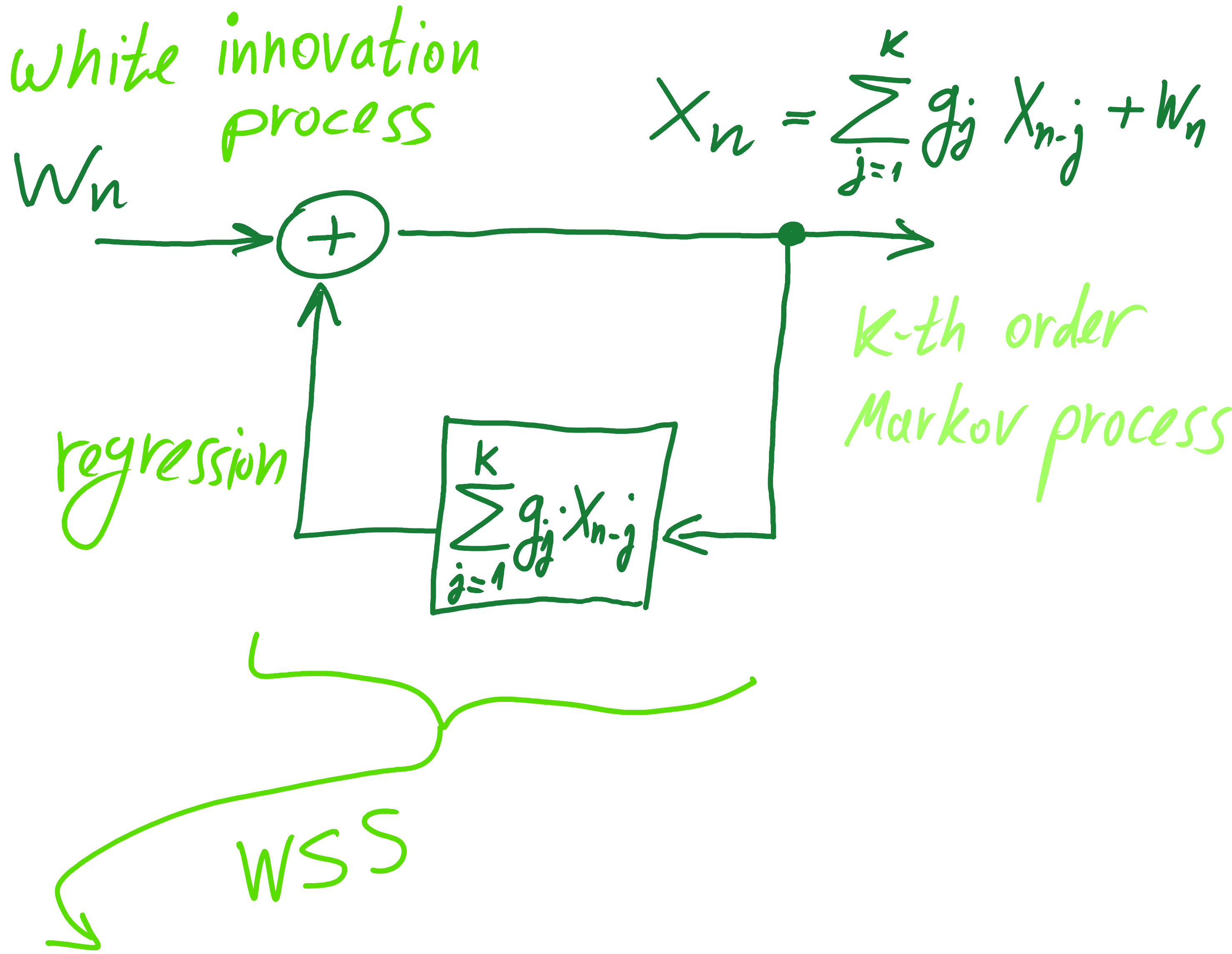

where $1 - \sum_{j=1}^{k} g_j \cdot z^{-j}$ is monic minimum phase (causally invertible)

Markov (auto-regressive) process

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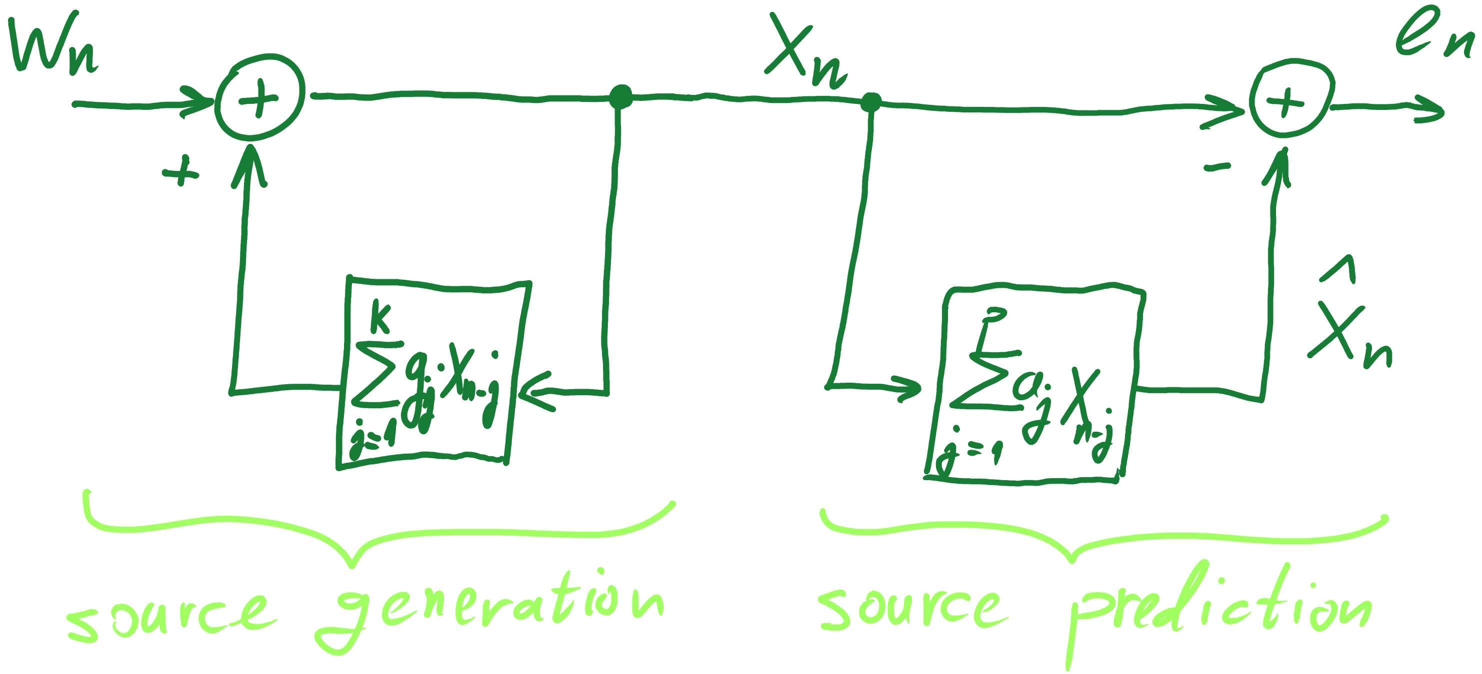
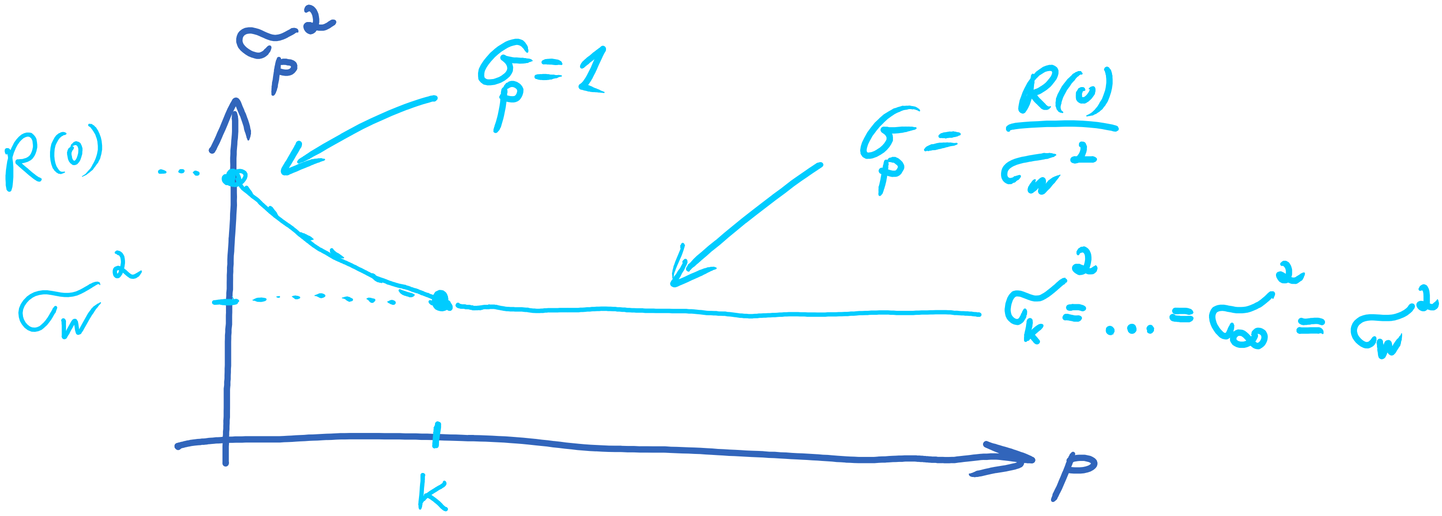


$$p \geq k \Rightarrow \underline{a}^{opt} = (\underbrace{g_1, \dots, g_k}_{k}, \underbrace{0 \dots 0}_{p-k})$$

$$\Rightarrow e_n = W_n = \text{white } !$$



Information & prediction

Saturday, April 15, 2017 6:10 PM

For Gaussian variable:

Variance ~ entropy

For Gaussian vectors:

condital variance ~ LMMSE

~ conditional entropy

Variance & entropy

Wednesday, March 08, 2017 2:07 PM

$$X_n \sim \mathcal{N}(0, \sigma_x^2)$$

$h(X_n)$ = differential entropy

$$\triangleq -\int_{-\infty}^{\infty} f_{X_n}(x) \log f_{X_n}(x)\, dx$$

$$= \frac{1}{2}\log\left(2\pi e\, \sigma_x^2\right)$$

$X_n \sim \mathcal{N}(0, \sigma_x^2)$

Joint entropy & covariance

Wednesday, April 19, 2017 8:43 AM

$$\underset{n\text{-dim}}{\underline{X}} \sim N(\underline{0}, \underset{n \times n}{\underline{\underline{R}}})$$

$$h(\underline{X}) = -\int_{\mathbb{R}^n} f_{\underline{X}}(\underline{x}) \log f_{\underline{X}}(\underline{x})\, d\underline{x}$$

$$= \frac{1}{2} \log\left((2\pi e)^n \det\{\underline{\underline{R}}\}\right)$$

Conditional variance & entropy

Wednesday, April 19, 2017 8:43 AM

$$\underline{X} \sim N(\underline{0}, \underline{\underline{R}})$$

$$f(x_n | x_{n-1}) = N(a^{opt} \cdot x_{n-1}, \sigma_1^2)$$

$$\vdots$$

optimal predictor

prediction error independent of past values

$$f(x_n | x_{n-1}, \dots, x_{n-p}) = N\left(\underline{a}^{opt} \cdot \begin{pmatrix} x_{n-1} \\ \vdots \\ x_{n-p} \end{pmatrix}, \sigma_p^2\right)$$

$\Longrightarrow$ conditional entropy via prediction error:

$$h(x_n | x_{n-1} \dots x_{n-p}) = \frac{1}{2} \log(2\pi e \sigma_p^2)$$

Entropy rates & prediction

Wednesday, March 08, 2017 2:07 PM

$$\bar{h} = \lim_{n\to\infty} \frac{1}{n} h(X_1, \dots, X_n) = \lim_{k\to\infty} h(X_0 \mid X_{-1}, \dots, X_{-k})$$

stationary process

Entropy rates & prediction

Wednesday, March 08, 2017 2:07 PM

$$\bar{h} = \lim_{n\to\infty} \frac{1}{n} h(X_1,\dots,X_n) = \lim_{k\to\infty} h(X_0 \mid X_{-1},\dots,X_{-k})$$

stationary process

Gaussian

$$\xrightarrow[k\to\infty]{} \frac{1}{2}\log\left(2\pi e\,\sigma_\infty^2\right)$$

Gaussian

$$\frac{1}{2}\log\left(2\pi e \sqrt[n]{\det\{\underline{\underline{R}}_n\}}\right) \xrightarrow[n\to\infty]{} \int_{-1/2}^{1/2} \frac{1}{2}\log\left(2\pi e\, S(f)\right) df$$

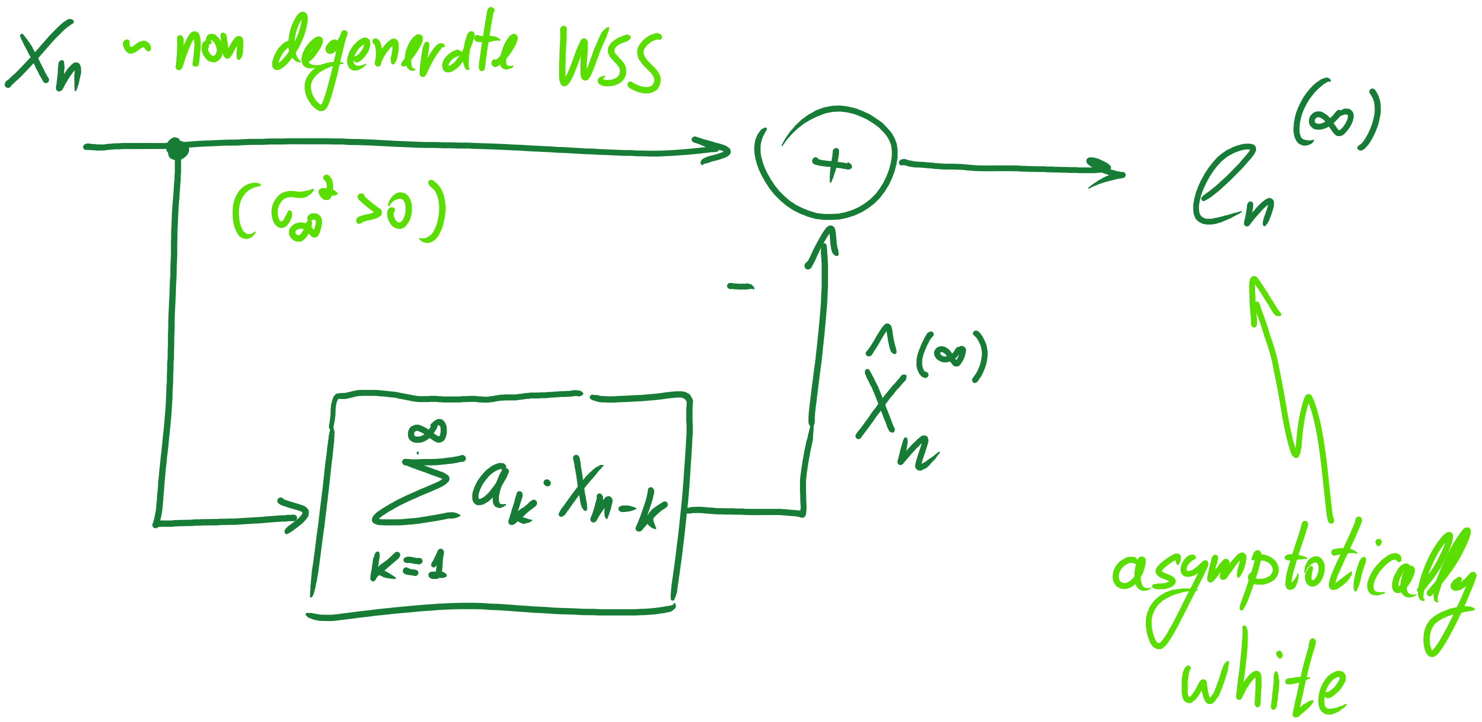
by frequency domain alysis

WSS

$$\Rightarrow \quad \sigma_\infty^2 = \exp \int_{-1/2}^{1/2} \log\left(S(f)\right) df$$

Infinite order prediction

Wednesday, March 08, 2017 1:50 PM

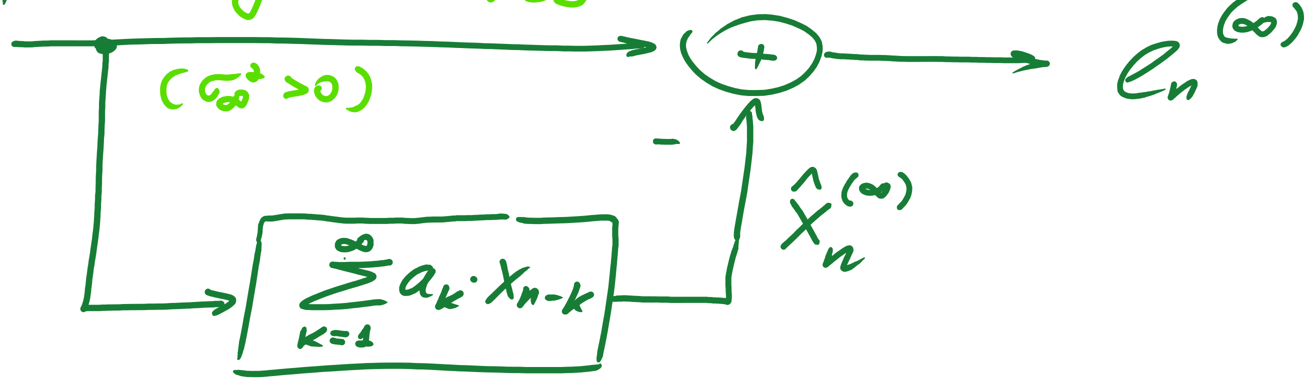


$$e_n^{(p)} \xrightarrow[p\to\infty]{} \text{white Gaussian noise } \mathcal{N}(0, \sigma_\infty^2)$$

Infinite order prediction

Wednesday, March 08, 2017 1:50 PM

x_n ~ non degenerate WSS



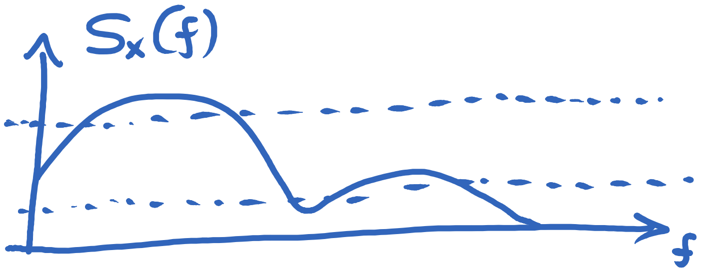
$$\sigma_\infty^2 = \exp\left\{ \int \log S_x(f)\, df \right\}$$

Gaussian

$$= \text{"geometric mean of spectrum"}$$

$$\simeq \sqrt[p]{\prod_{i=1}^{p} \lambda_i^{(p)}} \quad \text{as } p \to \infty$$

where $\lambda_1^{(p)}, \dots, \lambda_p^{(p)}$ = eigen values of $\underline{\underline{R}}_p$



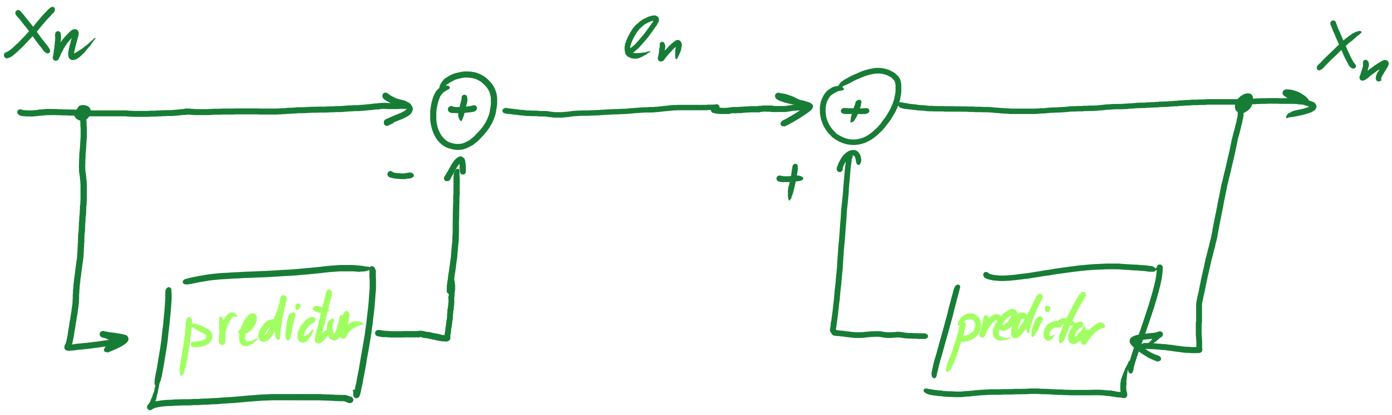
$R(0)$ = arithmetic mean

σ_∞^2 = geometric mean

$\therefore$ asymptotic prediction gain $= \dfrac{\sigma^2}{\sigma_\infty^2}$

Analysis & synthesis

Saturday, March 18, 2017 5:04 PM



whitening

= analysis

= FIR filter

coloring

= synthesis

= IIR filter

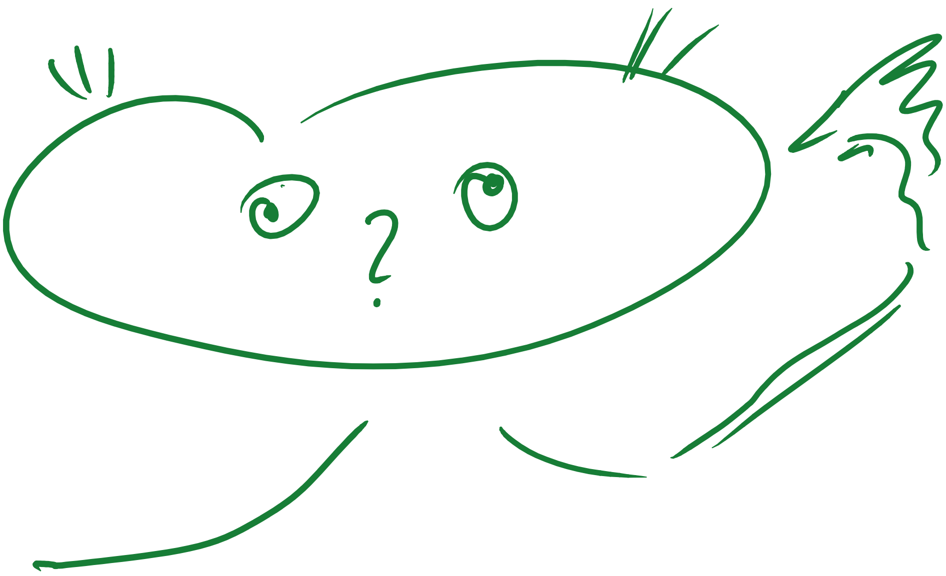
$$\left(\frac{out}{in}\right)^2_{FIR} = 1/G_p \quad \downarrow$$

$$\left(\frac{out}{in}\right)^2_{IIR} = G_p \quad \nearrow$$

Predictive Coding

Wednesday, April 19, 2017 9:45 AM

Can we use prediction for coding?



Source Coding

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Source $x_1 \dots x_n \longrightarrow \boxed{\text{encoder } f(\underline{x})} \xRightarrow{\text{digital message } W} \boxed{\text{decoder } g(w)} \longrightarrow$ reconstruction $y_1 \dots y_n$

$$W \in \{1, \dots, 2^{nR}\}$$

R = bit rate / source sample

---

lossless $\Rightarrow$ $y_n = x_n$ w.h.p.

lossy $\Rightarrow$ $MSE = \overline{E(y_n - x_n)^2}$

$$\boxed{\min_{f(\cdot), g(\cdot) \,:\, MSE \leq D} \{ R \} = ?}$$

Rate-Distortion Theory for Sources with Memory

Tuesday, March 07, 2017 12:27 PM

$$
R(D) = \begin{cases} \displaystyle\min_{\{y:\, E(y-x)^2 \le D\}} I(X;Y), & \text{memoryless} \\[2ex] \displaystyle\lim_{n\to\infty} \frac{1}{n} \min_{\{\underline{y}:\, \frac{1}{n} E\|\underline{y}-\underline{x}\|^2 \le D\}} I(\underline{X};\underline{Y}), & \text{memory} \end{cases}
$$

source $\quad$ reconstruction

$$X \longrightarrow p^*(y|x) = ? \longrightarrow Y$$

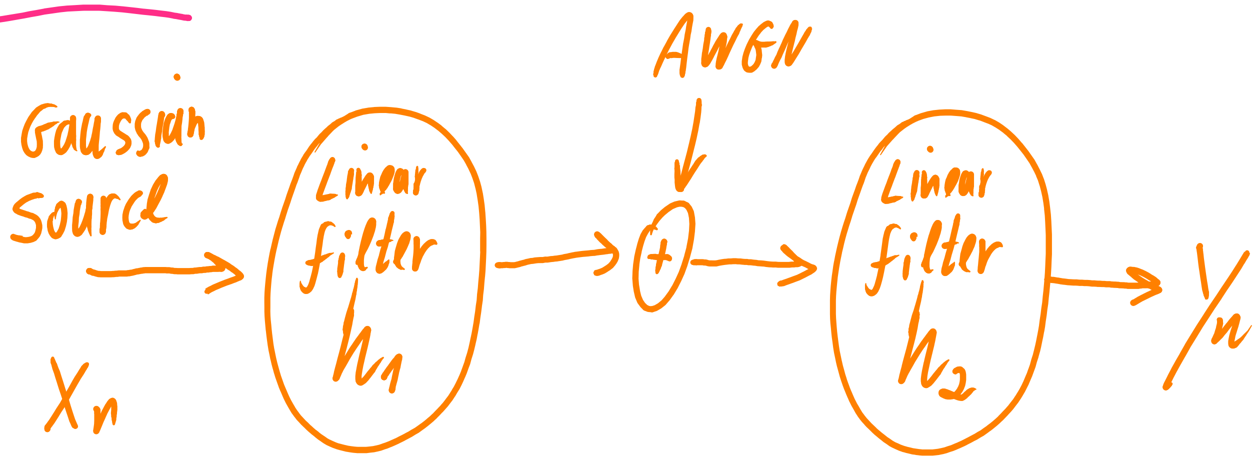
Gaussian source case (MSE distortion)

Tuesday, March 07, 2017 12:43 PM

- white source : $X_n \sim \mathcal{N}(0, \sigma_x^2)$ iid
- colored stat. source :
  - correlation $R_k \triangleq E\{X_n \cdot X_{n+k}\}$
  - spectrum $S_x(f) \triangleq \mathcal{F}\{R_k\}$

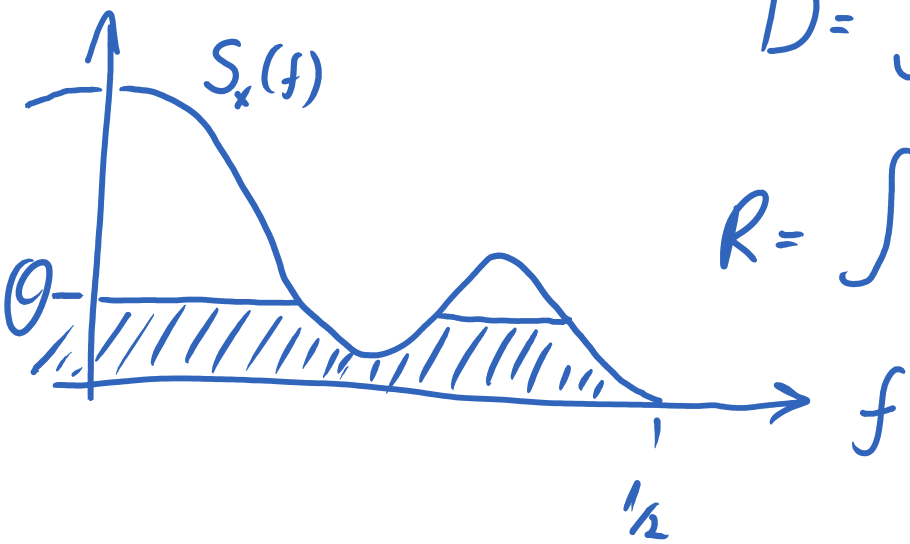
$$R(D) = \begin{cases} \frac{1}{2}\log\left(\frac{\sigma_x^2}{D}\right), & \text{white} \\ \int (\text{water-pouring})\, df, & \text{colored} \end{cases}$$

$p^*(y|x)$



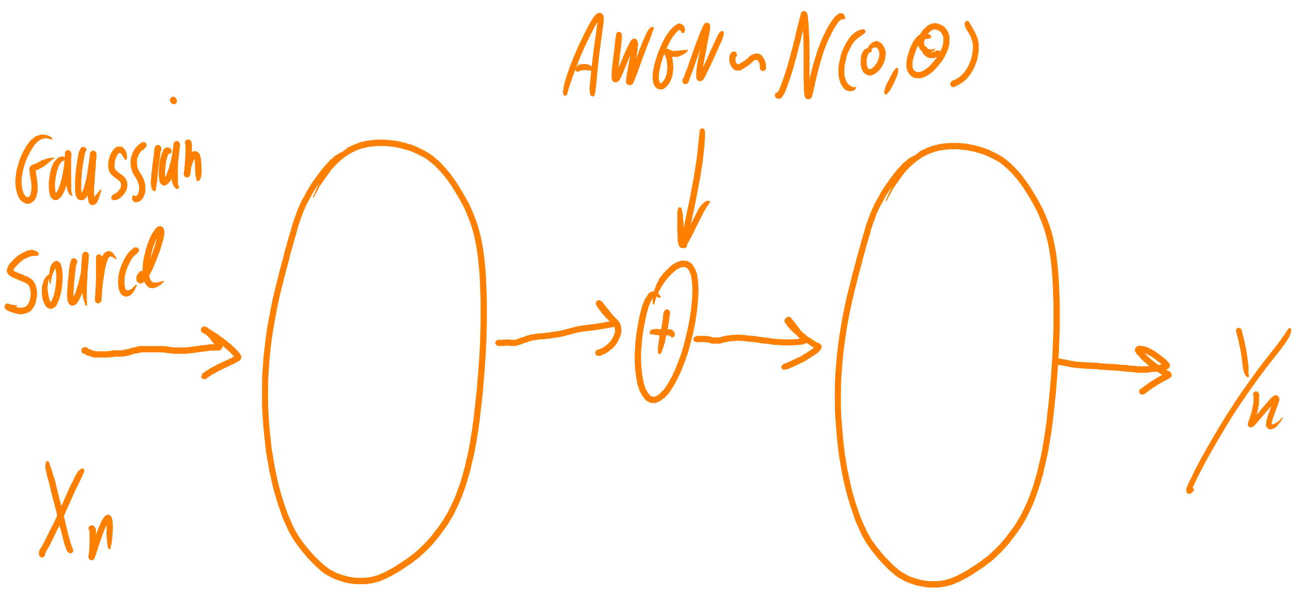
Water Pouring (frequency & time)

Wednesday, March 08, 2017 12:05 PM



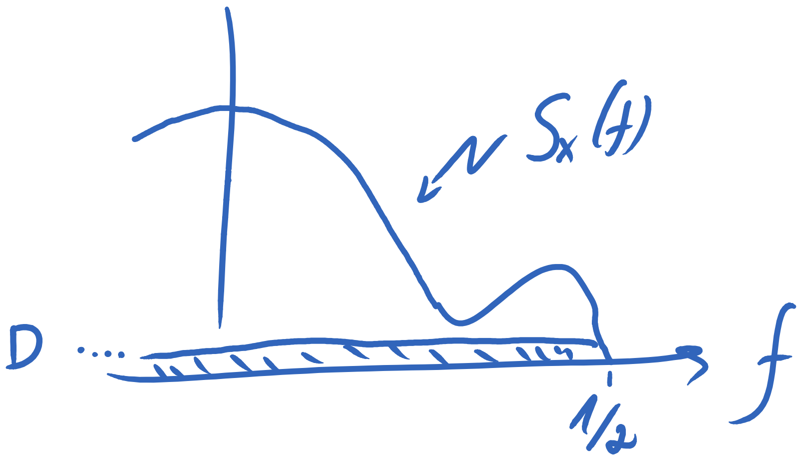
$$D = \int \min\{\theta,\ S_x(f)\}\, df$$

$$R = \int \frac{1}{2} \cdot \log^+\left(\frac{S_x(f)}{\theta}\right) df$$



High Signal-to-Distortion

Saturday, March 18, 2017 5:17 PM



$$D \leq S_{min}(f)$$

$$R(D) = \frac{1}{2}\log\left(\frac{\sigma_\infty^2}{D}\right)$$

where

$$\sigma_\infty^2 = \text{entropy power} = \exp \int \log S_x(f)\, df$$

$$= \text{geometric mean of } S_x(f)$$

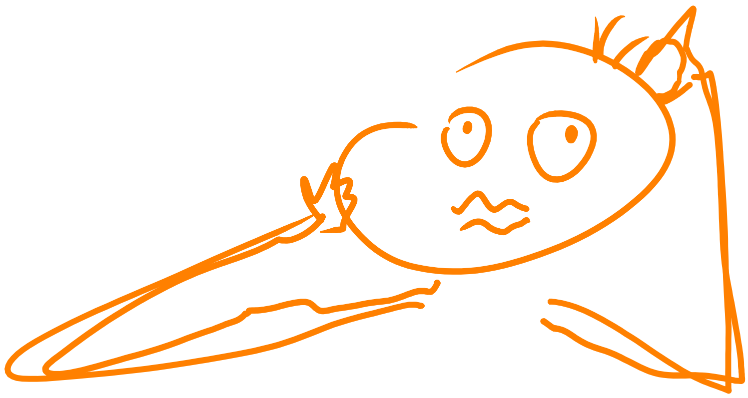
$\Rightarrow$ memory gain $= \sigma_x^2 / \sigma_\infty^2$ = prediction gain !

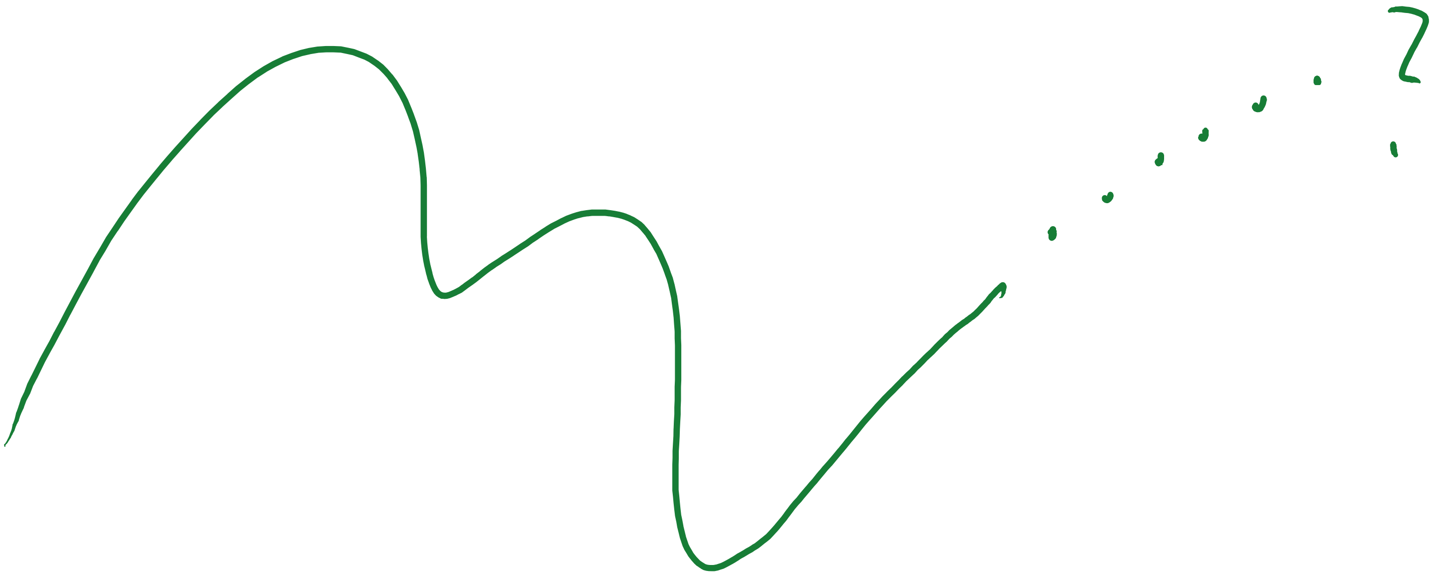
$$\therefore \; R_{colored}(D) = R_{white}(D) - \frac{1}{2}\log(G_p)$$

Predictive Coding

Saturday, March 18, 2017 4:13 PM

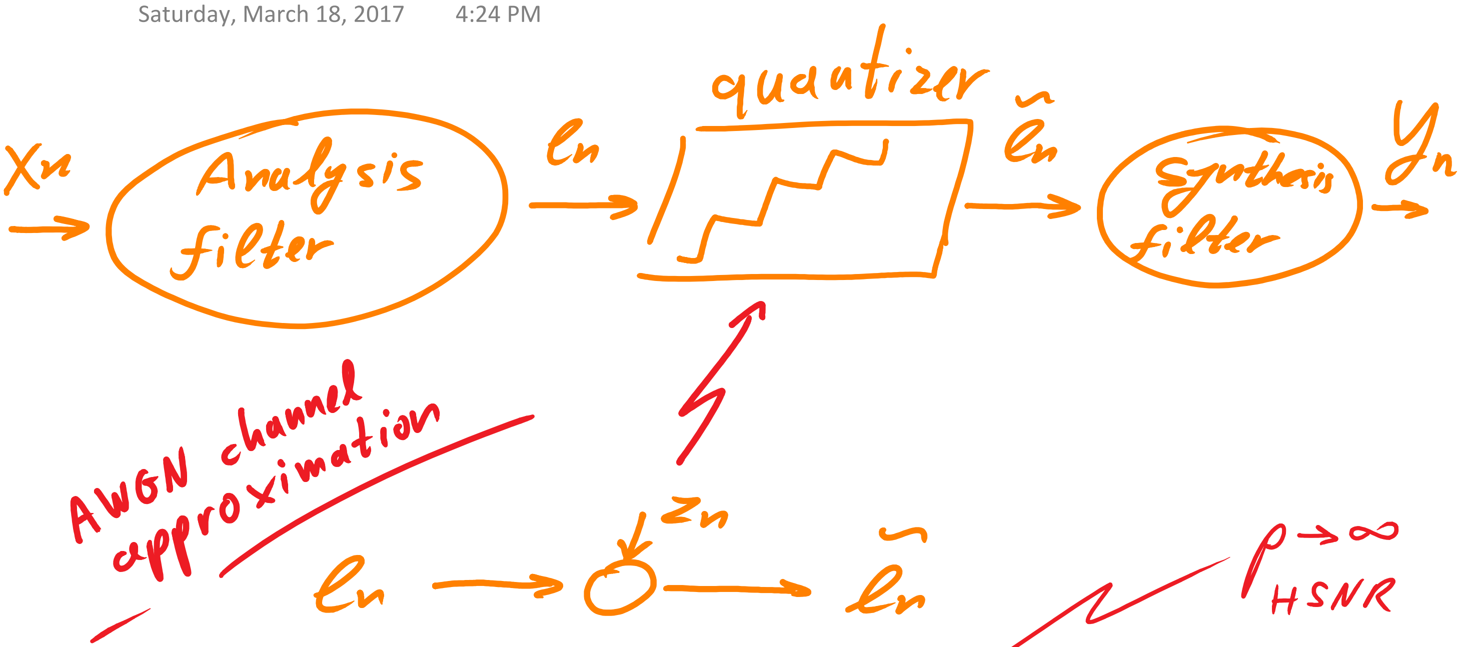
Can we exploit source memory by prediction?





How *not* to do it...

Saturday, March 18, 2017 4:24 PM



$$R = I(e_n; e_n + z_n) \approx \frac{1}{2}\log\left(\frac{\sigma_\infty^2}{\sigma_z^2}\right)$$

$$\begin{aligned} y_n &= \text{synthesis} * \tilde{e}_n \\ &= \text{synthesis} * (e_n + z_n) \\ &= x_n + \text{synthesis} * z_n \end{aligned}$$

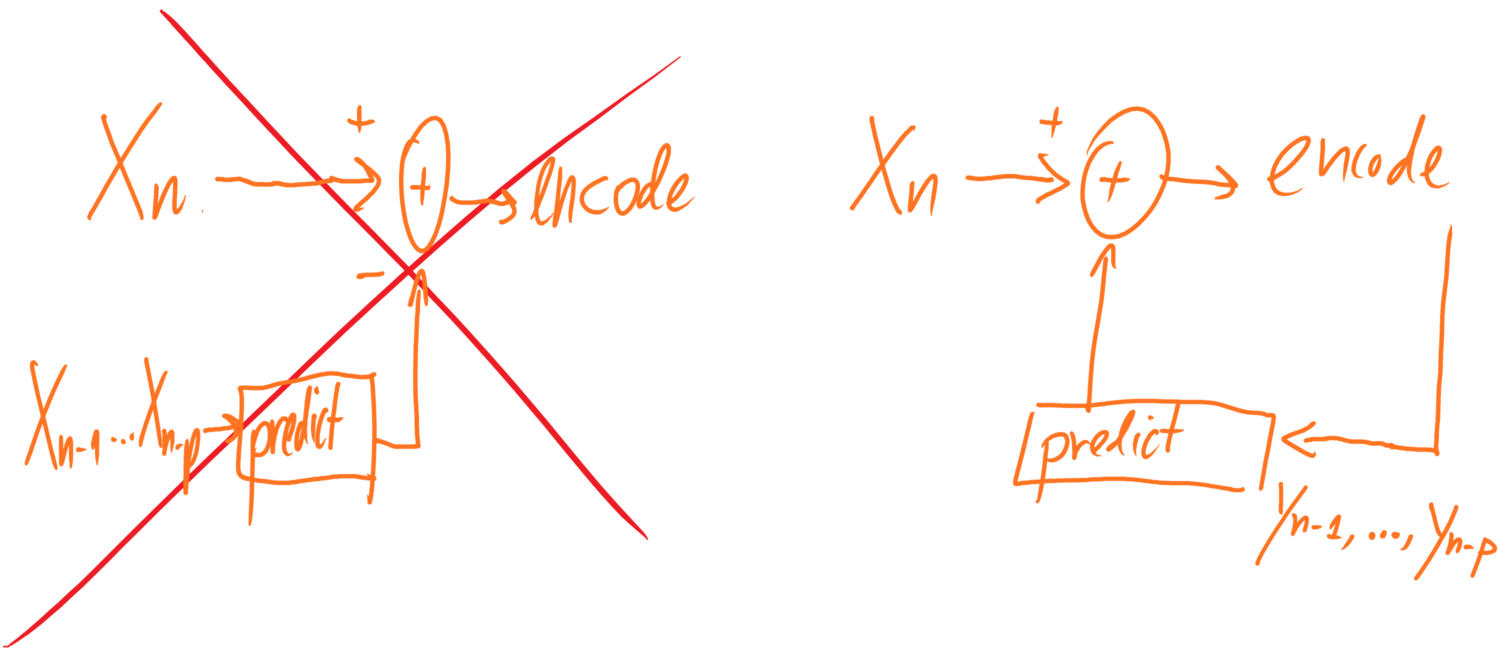
$$\begin{aligned} D = E(y_n - x_n)^2 &= E(\text{synthesis} * z_n)^2 \\ &= G_p \cdot \sigma_z^2 = \frac{\sigma_x^2}{\sigma_\infty^2} \cdot \sigma_z^2 \end{aligned}$$

$$\Rightarrow \quad R = \frac{1}{2}\log\left(\sigma_x^2 / D\right) = \text{as if white}$$

Predictive coding: the right way!

Saturday, March 18, 2017 5:23 PM

do it in closed loop!

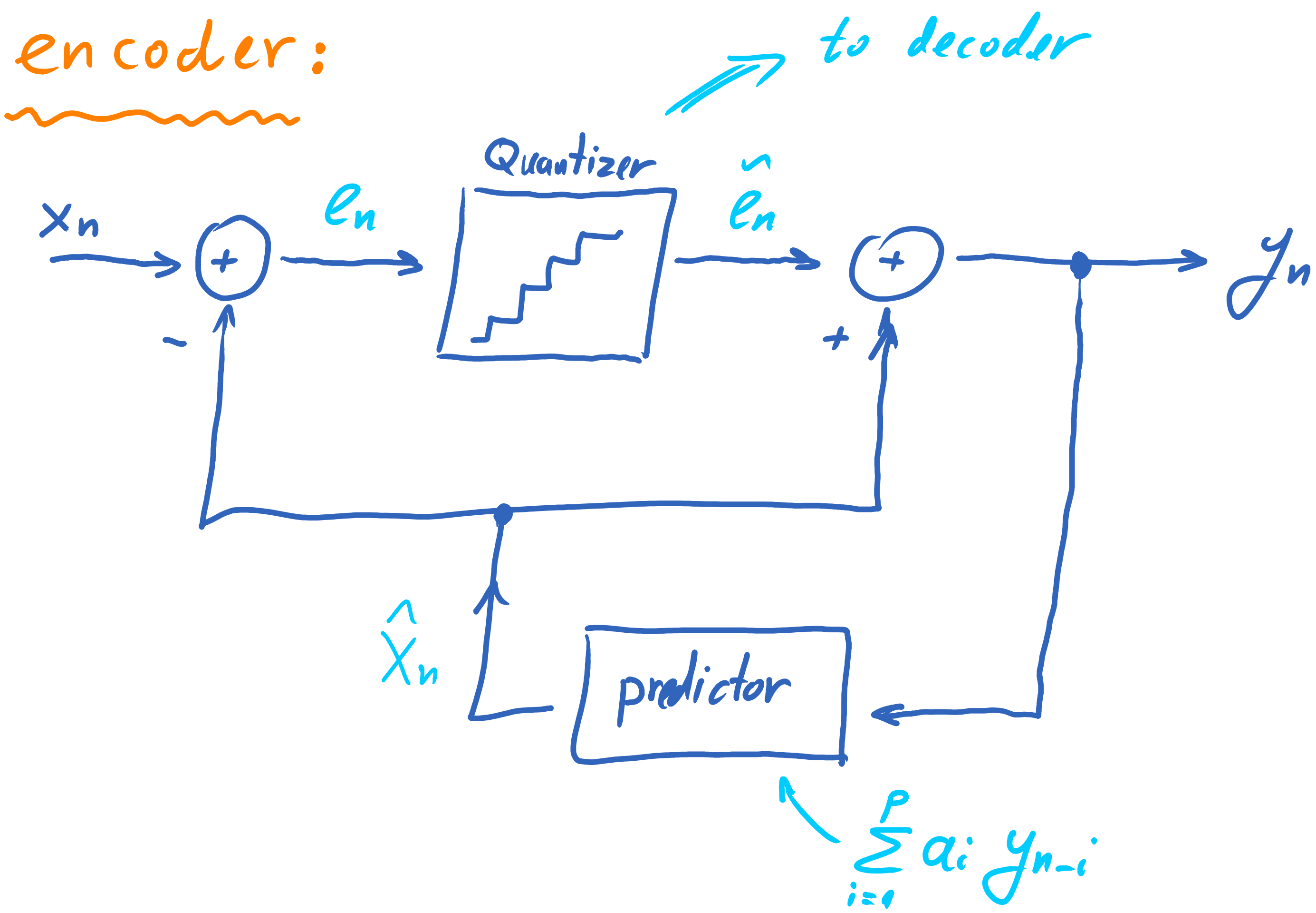


$\Rightarrow$ Differential Pulse-Code Modulation

(DPCM)

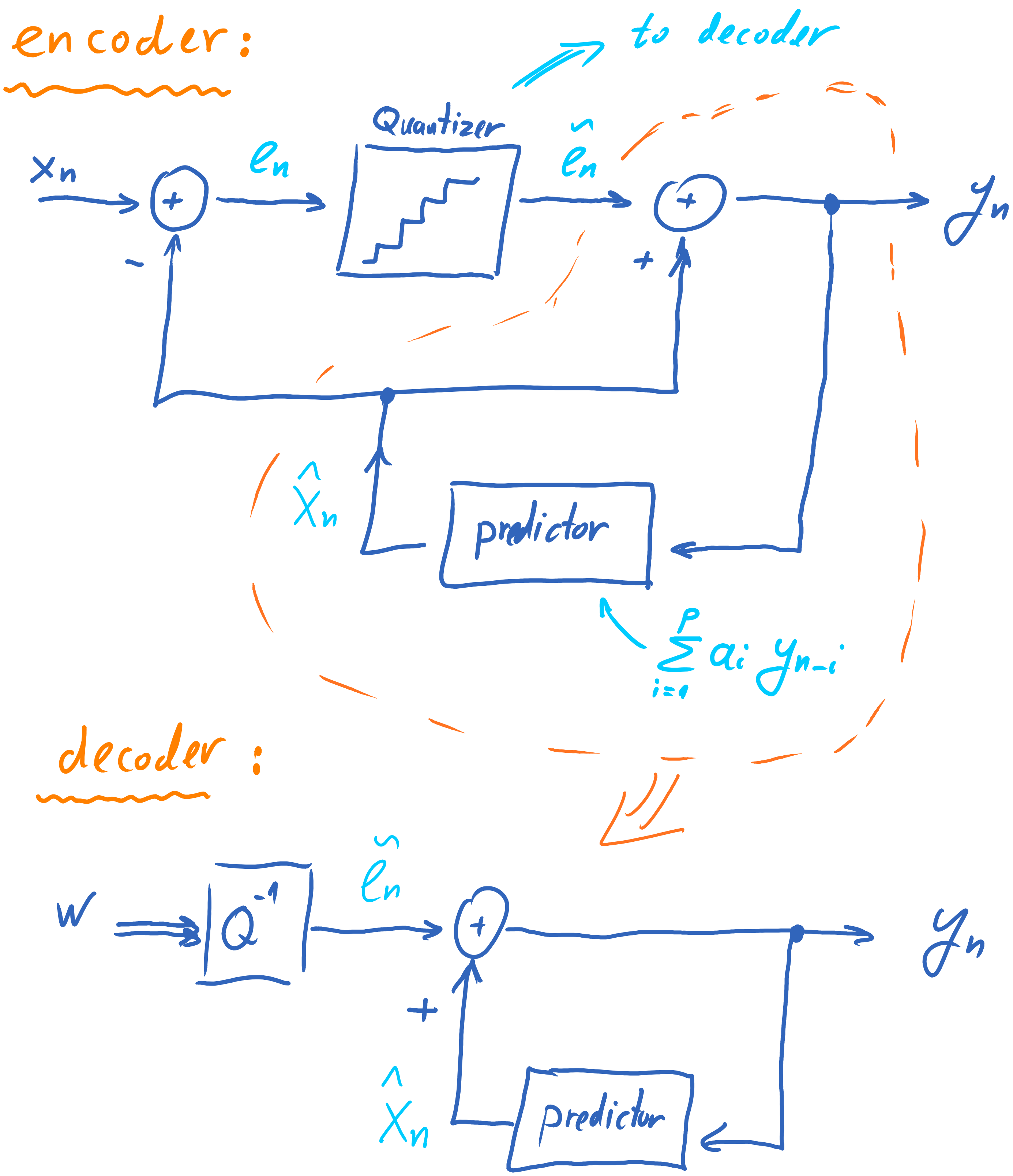
Differential Pulse Code Modulation

Wednesday, April 19, 2017 11:09 AM



Differential Pulse Code Modulation

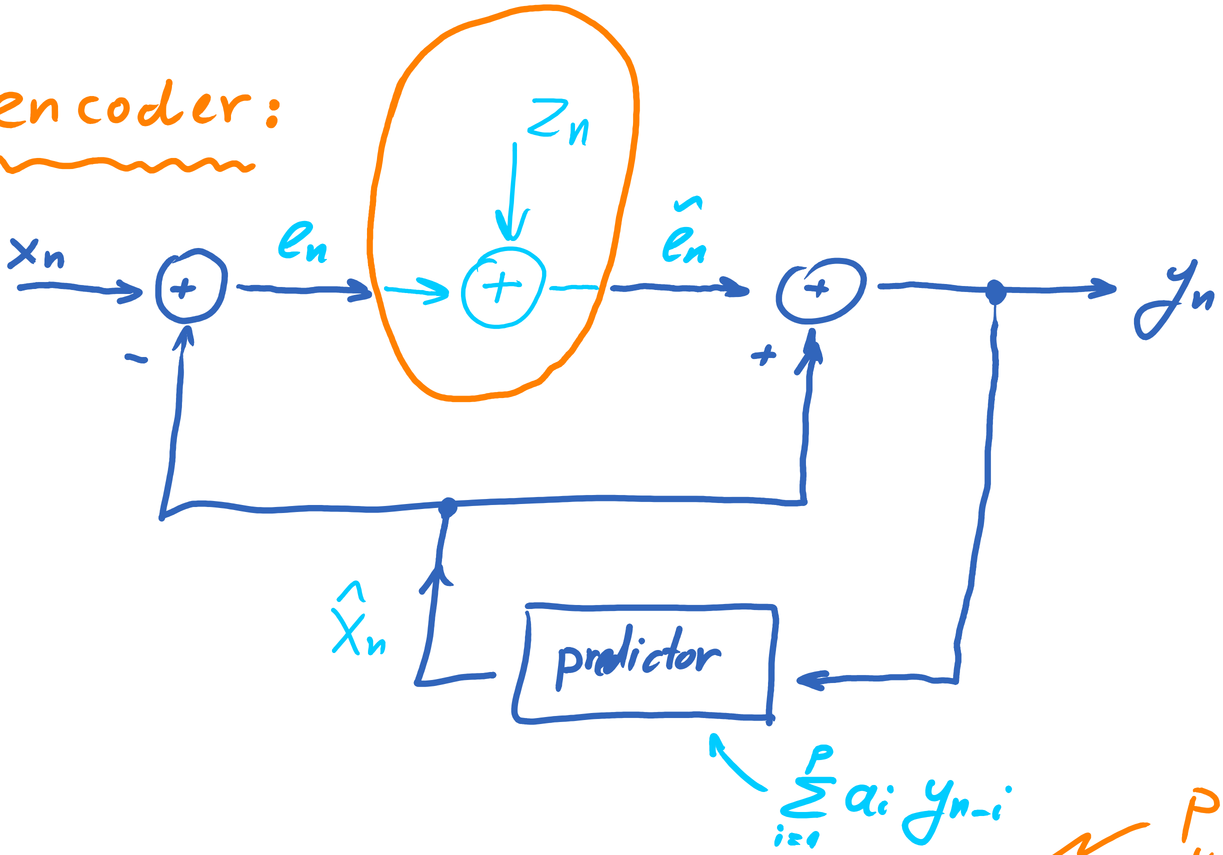
Wednesday, April 19, 2017 11:09 AM



DPCM: analysis

Wednesday, April 19, 2017 11:09 AM

encoder:



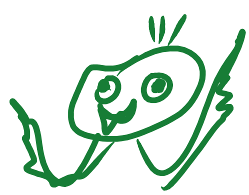
? $p \to \infty$ HSNR

$$R = I(e_n ; e_n + z_n) \simeq \frac{1}{2} \log \left(\frac{\sigma_e^2}{\sigma_z^2} \right)$$

$$y_n = x_n - \cancel{\hat{x}_n} + z_n + \cancel{\hat{x}_n} = x_n + z_n$$

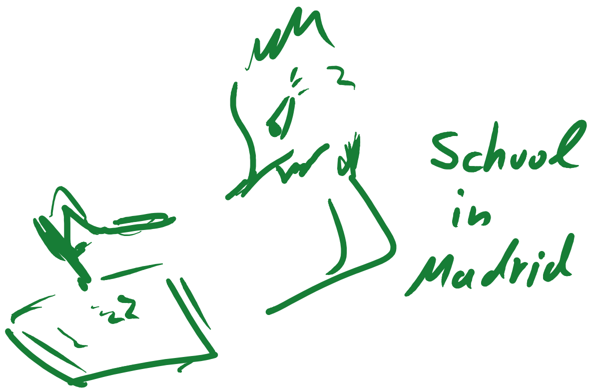
$$D = E\{(y_n - x_n)^2\} = E\{(\hat{e}_n - e_n)^2\} = E(z_n^2) = \sigma_z^2$$

$$\Rightarrow R = \frac{1}{2} \log \left(\frac{\sigma_\infty^2}{D} \right) = R(D) \ !$$



Homework!

Saturday, May 06, 2017 7:31 PM



$$\sigma_\infty^2(x_n \mid \underbrace{y_{n-1}, y_{n-2} \dots}_{\text{noisy past}}) \overset{?}{\leq} \sigma_\infty^2(x_n \mid \underbrace{x_{n-1}, x_{n-2}, \dots}_{\text{clean past}})$$

* general relation ?

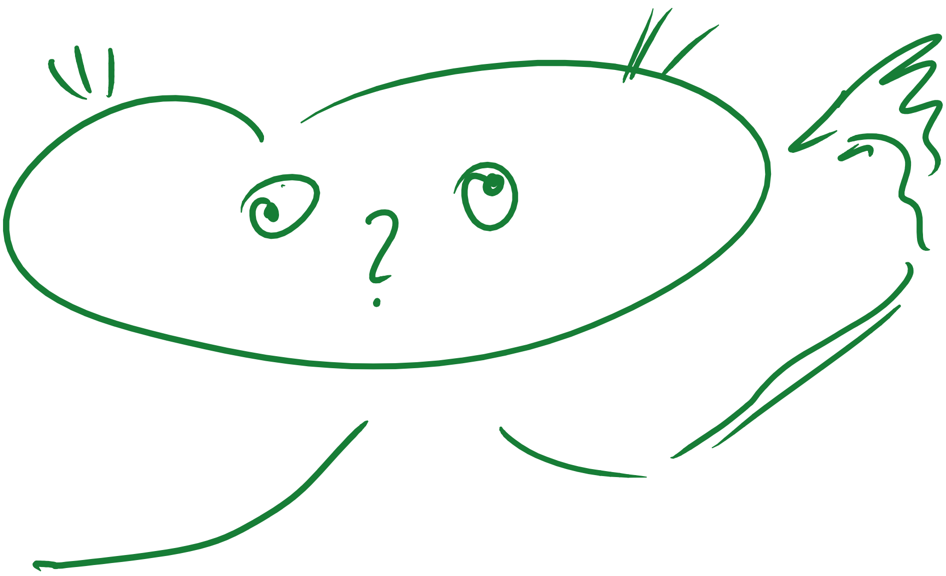
* $\sigma_\infty^2(y_n \mid y_{n-1}, y_{n-2} \dots) = ?$

* $\underline{a}^{opt}(x_{n-1} \mid y_{n-1}, \dots, y_{n-p}) = ?$

Prediction for Channels

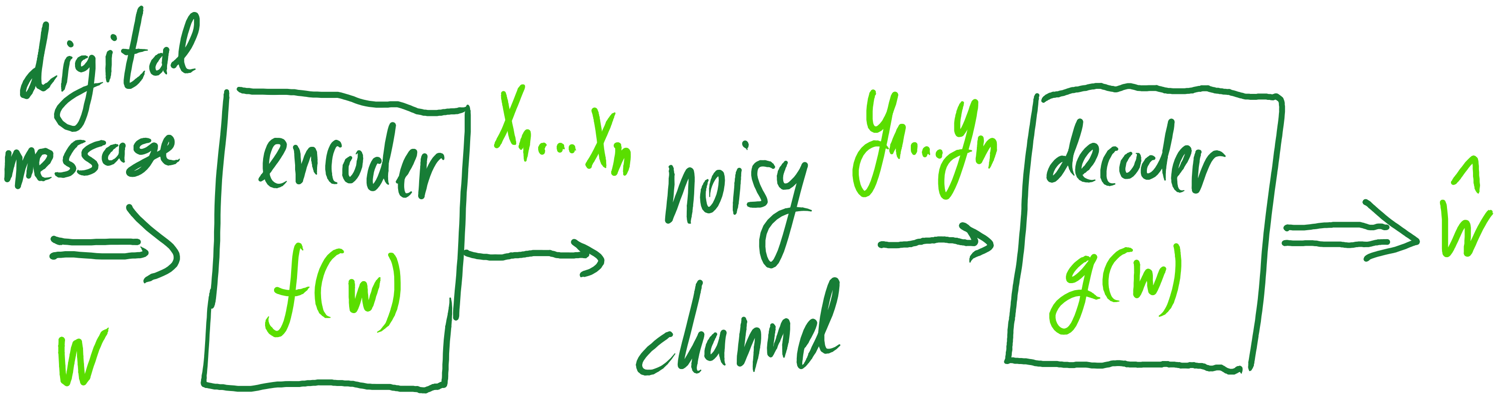
Wednesday, April 19, 2017 9:45 AM

Can we do the same for channels ?



Channel Coding

Saturday, April 22, 2017 2:51 PM



$$W \in \{1, \dots, 2^{nR}\}$$

$R =$ bits / channel use

$$P_e = Pr(\hat{W} \neq W)$$

$$\max \{ R \} = \, ?$$

$f(\cdot), g(\cdot)$: power $\leq P$

$P_e \to 0$

Channel Coding Theorem (Shannon 1948)

Saturday, April 22, 2017 2:58 PM

$$\underline{x} \longrightarrow \underset{\text{channel}}{\boxed{p(\underline{y}|\underline{x})}} \longrightarrow \underline{y}$$

* The maximal "achievable" rate $\underline{R}$ is the capacity $\underline{C}$:

$$C = \begin{cases} \max\limits_{\{p(x):\, EX^2 \le P\}} I(X;Y), & \underline{\text{memoryless}} \\[2em] \lim\limits_{n\to\infty} \frac{1}{n} \max\limits_{\{p(\underline{x}):\, \frac{1}{n}E\|\underline{X}\|^2 \le P\}} I(\underline{X};Y), & \underline{\text{memory}} \end{cases}$$

* $p^*(x)$ = optimal input distribution = arg max = ?

Gaussian Channels

Sunday, April 23, 2017 9:14 AM

* AWGN channel (memoryless)

input power constraint $EX^2 \le P$

$Z \sim N(0, \sigma_z^2)$ iid

$$X \longrightarrow \oplus \longrightarrow Y$$

* Filter (intersymbol interference) channel

$Z_n \sim$ AWGN

$$X_n \longrightarrow \boxed{H(z)} \longrightarrow \oplus \longrightarrow Y_n$$

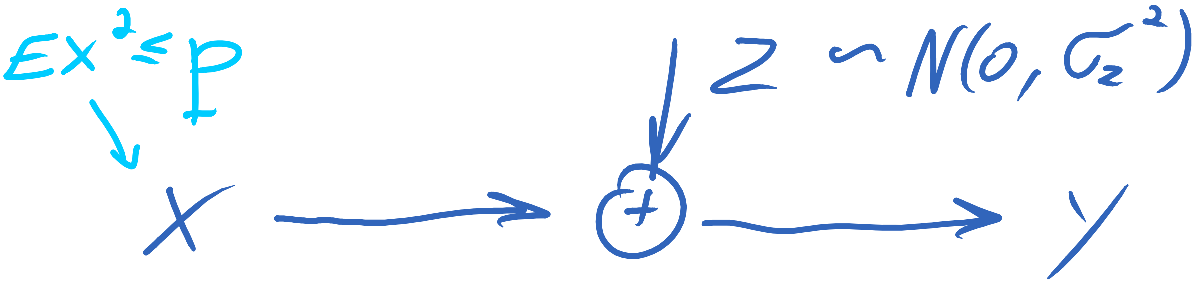
$$H(z) \triangleq 1 + \sum_{i=1}^{\infty} h_i \cdot z^{-i}$$

$$\Rightarrow y_n = X_n + \sum_{k=1}^{\infty} h_k \cdot X_{n-k} + Z_n$$

Gaussian Channels

Sunday, April 23, 2017 9:14 AM

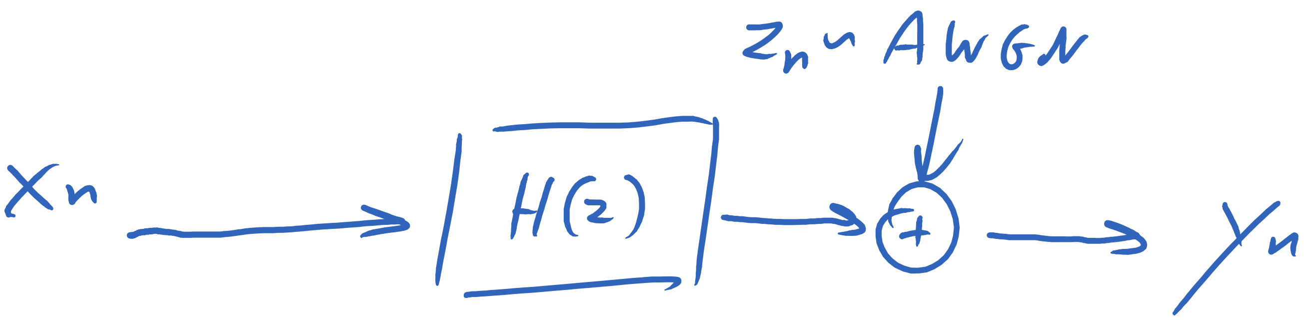
* **AWGN channel (memoryless)**



$$\Rightarrow p^*(x) = N(0, P), \quad C = \frac{1}{2}\log(1 + SNR)$$

$SNR = P/\sigma_z^2$

* **Filter (intersymbol interference) channel**

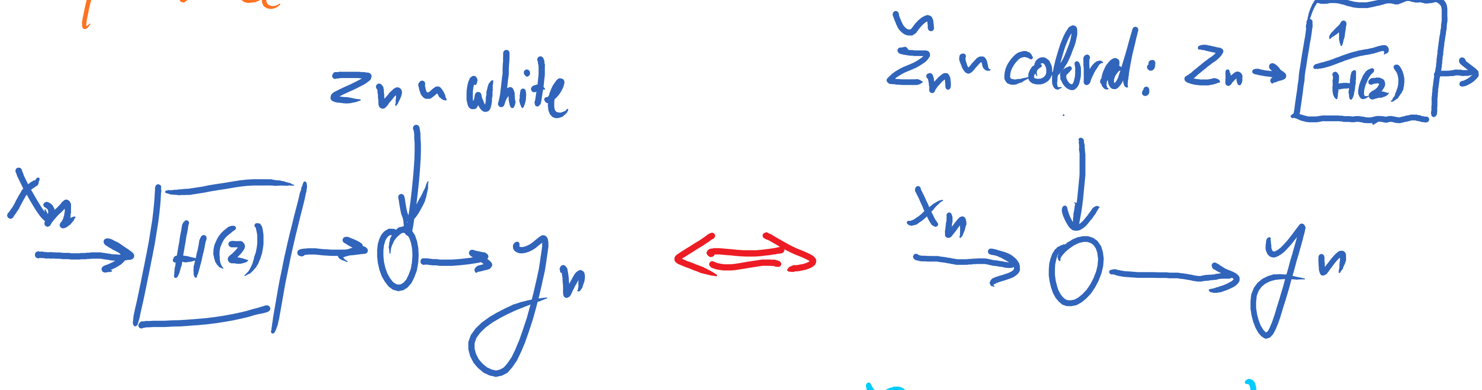


$C =$ "water pouring"

Water Pouring for channels

Sunday, April 23, 2017 9:37 AM

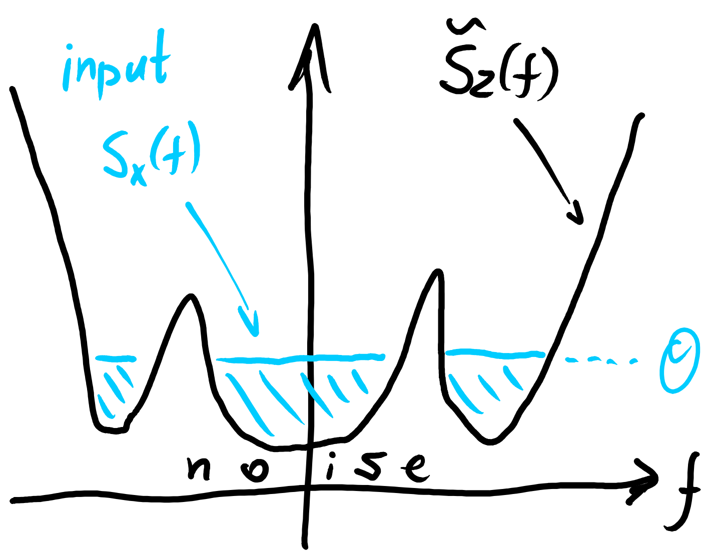
* equivalent channel models:



$S_z(f) = \sigma^2 \;\; \forall f$

$\tilde{S}_z(f) = \dfrac{\sigma^2}{|H(e^{j2\pi f})|^2}$

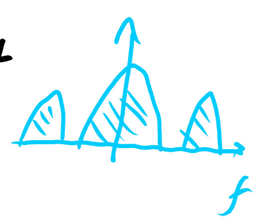
* water pouring solution:



$$P = \int \left[\theta - \tilde{S}_z(f)\right]^+ df$$

$$C = \int \left[\log\left(\frac{\theta}{\tilde{S}_z(f)}\right)\right]^+ df$$

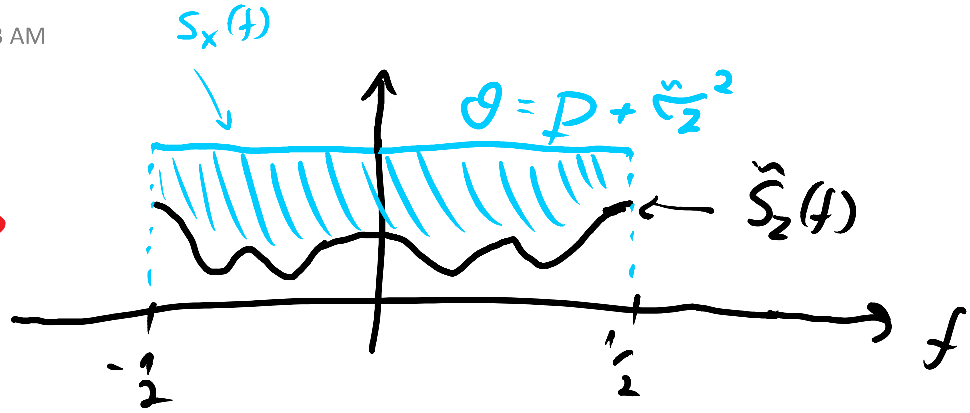
$S_x^*(f)$ = optimum input spectrum $= \left[\theta - \tilde{S}_z(f)\right]^+$



High SNR

Sunday, April 23, 2017 10:03 AM

$$P + \tilde{\sigma}_z^2 \geq \hat{S}_{z\,max} \Rightarrow$$



$$\Rightarrow \quad C = \int \frac{1}{2} \log\left(\frac{P + \tilde{\sigma}_z^2}{\hat{S}_z(f)}\right) df$$

$$\stackrel{\sim}{=} \frac{1}{2} \log\left(\frac{P + \cancel{\tilde{\sigma}_z^2}}{\tilde{\sigma}_\infty^2}\right)$$

$P \gg \tilde{\sigma}_z^2$

where $\tilde{\sigma}_\infty^2 = \exp \int \log \hat{S}_z(f)\, df$

$=$ noise prediction error

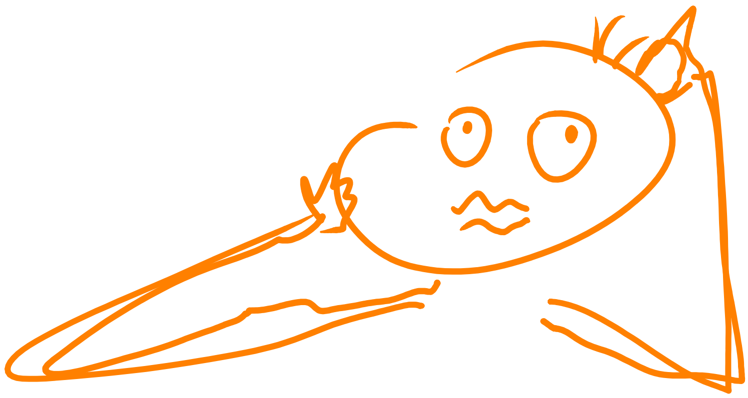
$$\Rightarrow C_{colored} = C_{AWGN} + \frac{1}{2} \log(G_p) \quad !$$

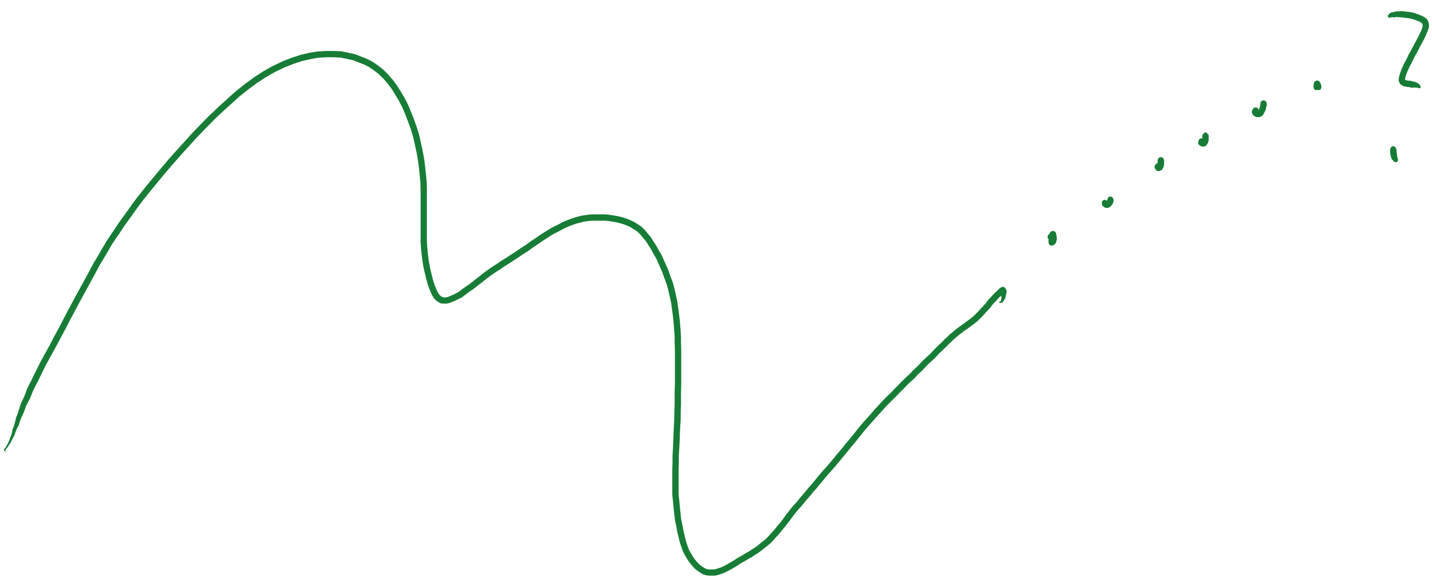
$\sim \frac{1}{2} \log\left(\frac{P}{\tilde{\sigma}_z^2}\right)$ $\qquad \frac{\tilde{\sigma}_z^2}{\tilde{\sigma}_\infty^2}$

Prediction & Information 2

Saturday, March 18, 2017 4:13 PM

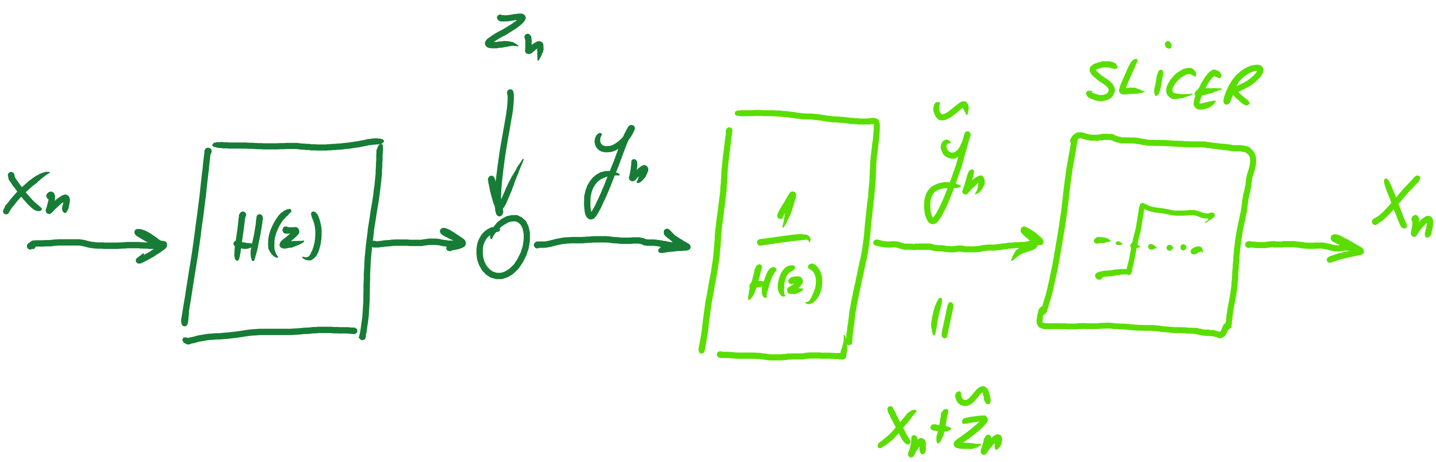
How can we exploit channel memory by prediction ?





The wrong way...

Sunday, April 23, 2017 11:59 AM



$$R_{max} \hat{=} I(\text{slicer}) = I(x_n ; x_n + \hat{\tilde{z}}_n) =$$

coding & detection

$$= \frac{1}{2}\log(1 + \text{SNR @ slicer})$$

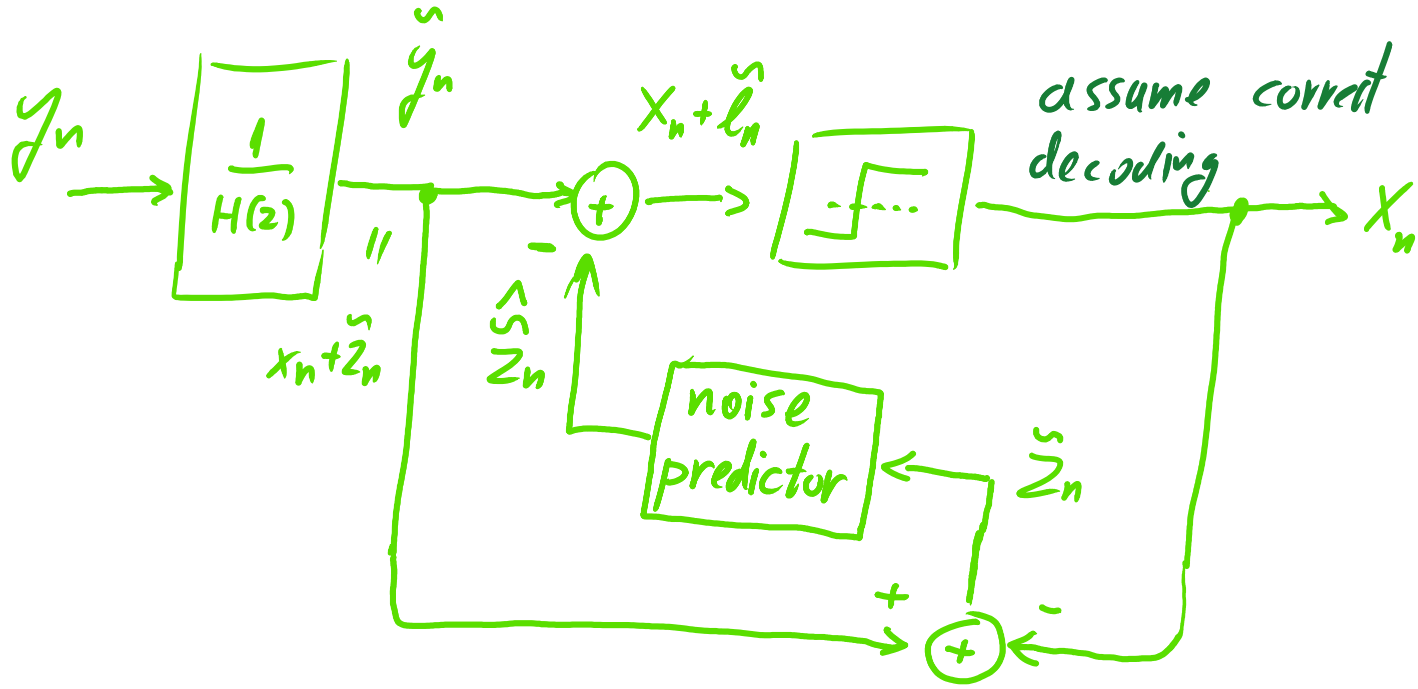
$$= C_{AWGN}\left(\text{SNR} = \frac{P}{\tilde{\sigma}_z^2}\right)$$

$\Rightarrow$ no prediction gain

The right way!

Sunday, April 23, 2017 2:07 PM

Noise-prediction equalizer:



$$R \overset{\sim}{=} I(\text{slicer}) = I(x_n ; x_n + e_n)$$

decoding & detection

$$= \frac{1}{2} \log\left(1 + \text{SNR @ slicer}\right)$$

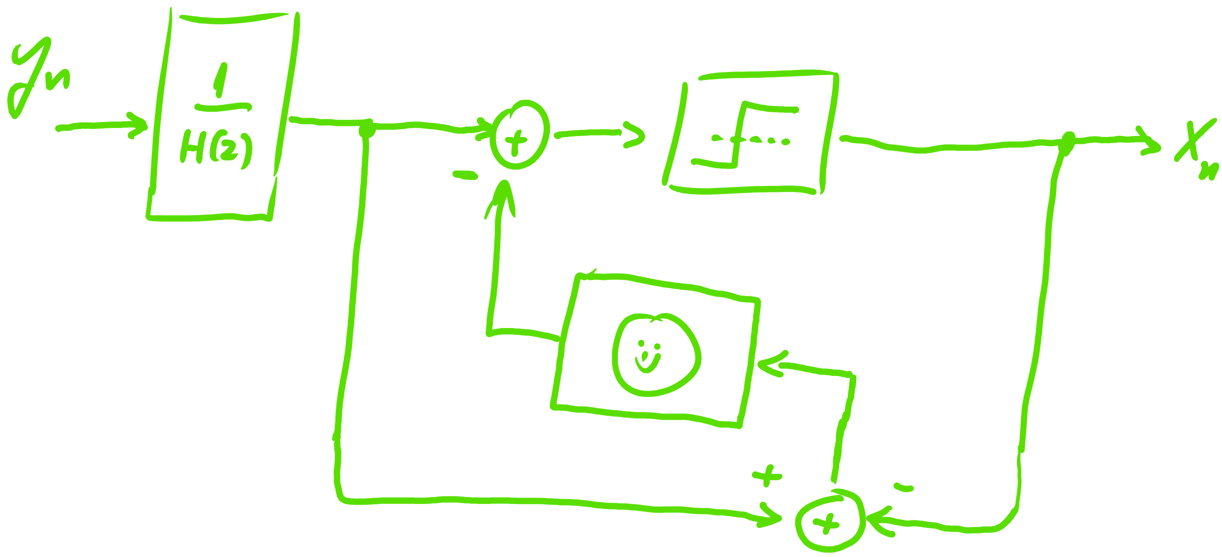
$$\overset{\sim}{=} \frac{1}{2} \cdot \log\left(\frac{P}{\tilde{\sigma}_\infty^2}\right)$$

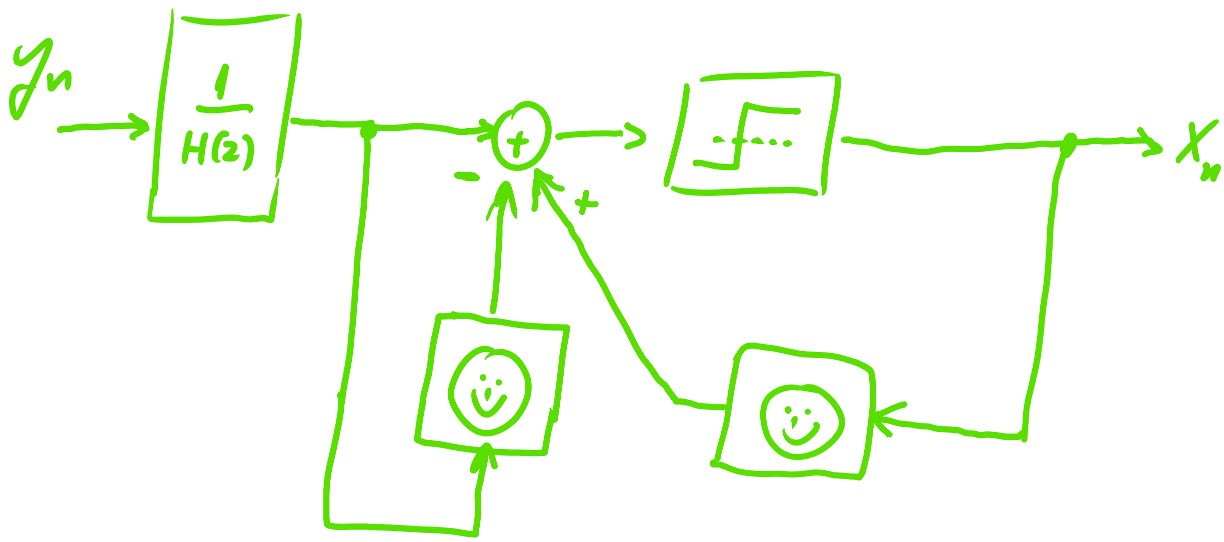
HSNR

$$= C_{AWGN} + \frac{1}{2} \log\left(G_p\right)$$

Equivalent form..

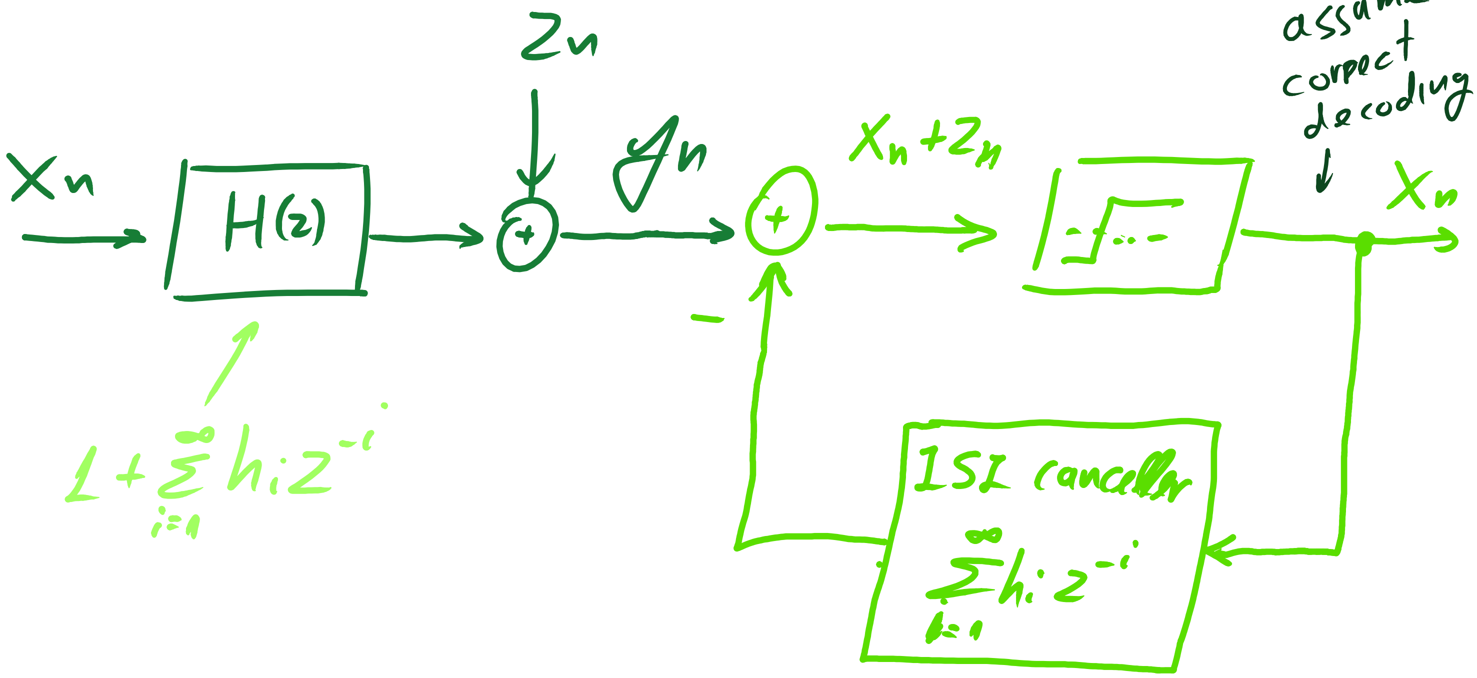
Sunday, April 23, 2017 2:07 PM





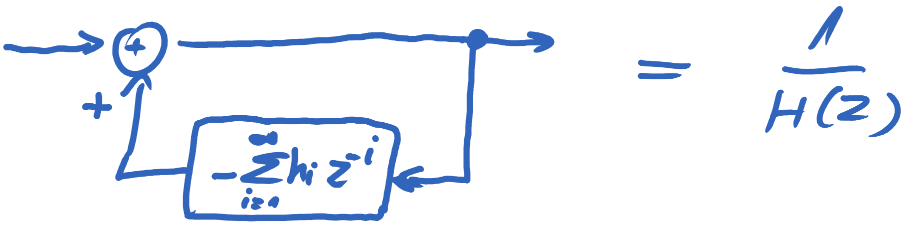
Decision-Feedback Equalizer

Sunday, April 23, 2017 2:23 PM

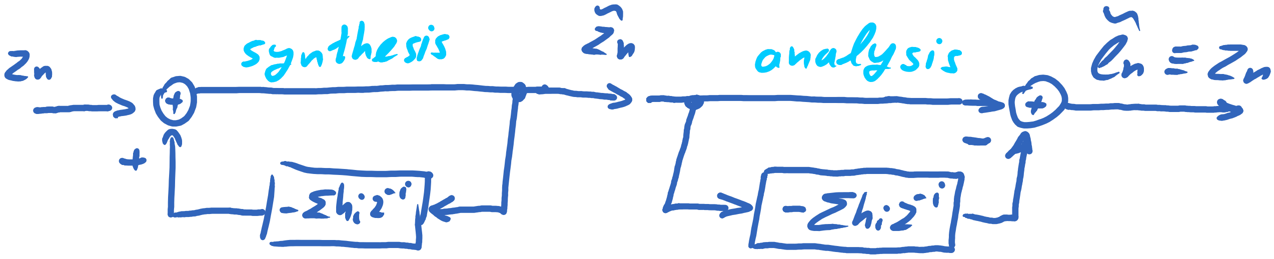


$$R_{max} \hat{=} I(\text{slicer}) = \dots \hat{=} \frac{1}{2}\log\left(\frac{P}{\sigma_z^2}\right) = \frac{1}{2}\log\left(\frac{P}{\tilde{\sigma}_\infty^2}\right) = C$$

* because



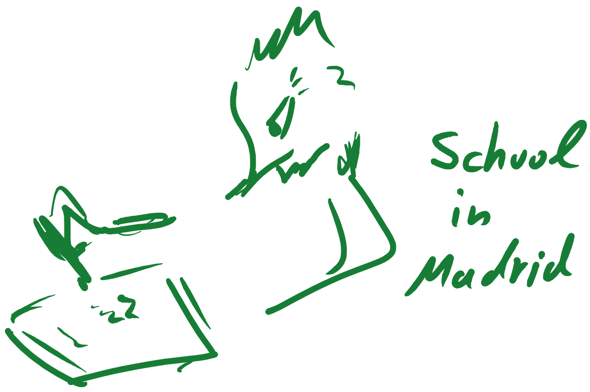
⇒

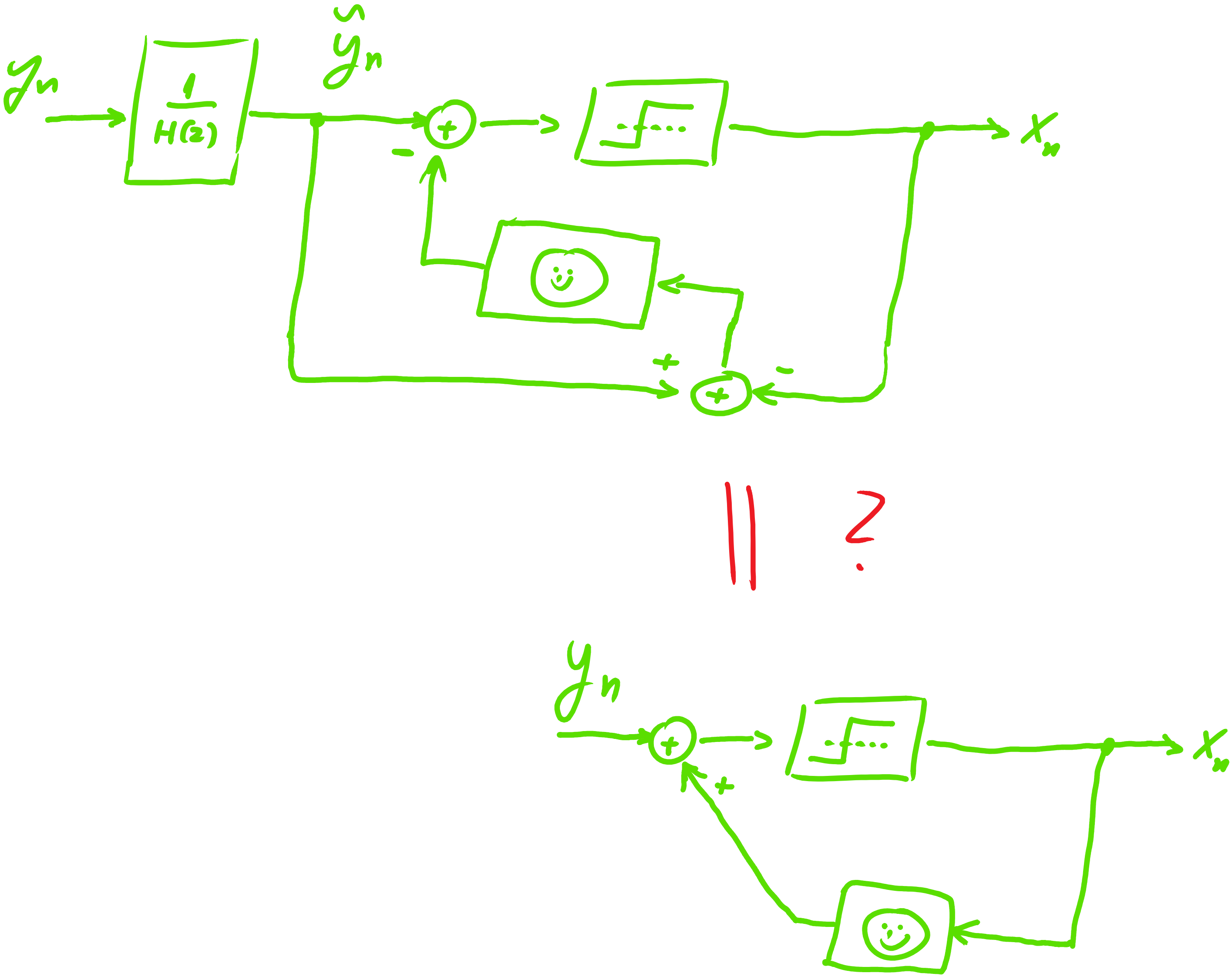


⇒ $\tilde{\sigma}_\infty^2$ = prediction error of $\tilde{z}_n$ = σ_z^2

Homework 2

Saturday, May 06, 2017 7:31 PM





* Which one do we prefer?

Information & Prediction: Summary

Sunday, April 23, 2017 5:21 PM

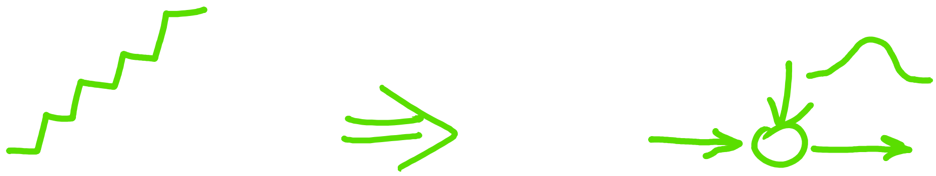
- Colored RDF ⇒ scalar mutual information after prediction
- Colored Capacity ⇒ scalar mutual information on slicer (after prediction)
- Entropy (given past) ⇒ Prediction error
- High SNR approximations

Still open...

Wednesday, April 26, 2017 8:18 AM

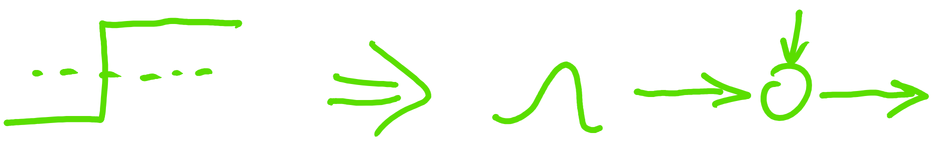
* How to relate ... ?

quantization → mutual information over AWGN channel



* How to relate ... ?

slicer error → mutual information @ Gaussian input



* Non high-SNR approximations ?