Disjoint LDPC Coding for Gaussian Broadcast Channels

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Introduction

- **Goal:** looking for a practical coding method for a two-user Gaussian broadcast channel

- **Broadcast channel:** simultaneous communication of a single source with multiple receivers

- Bergsman theorem: for some $p(x, v)$, capacity region of a degraded BC is

$$R_z \leq I(V; Z), \quad R_y \leq I(X; Y|V)$$

- Gaussian broadcast channel

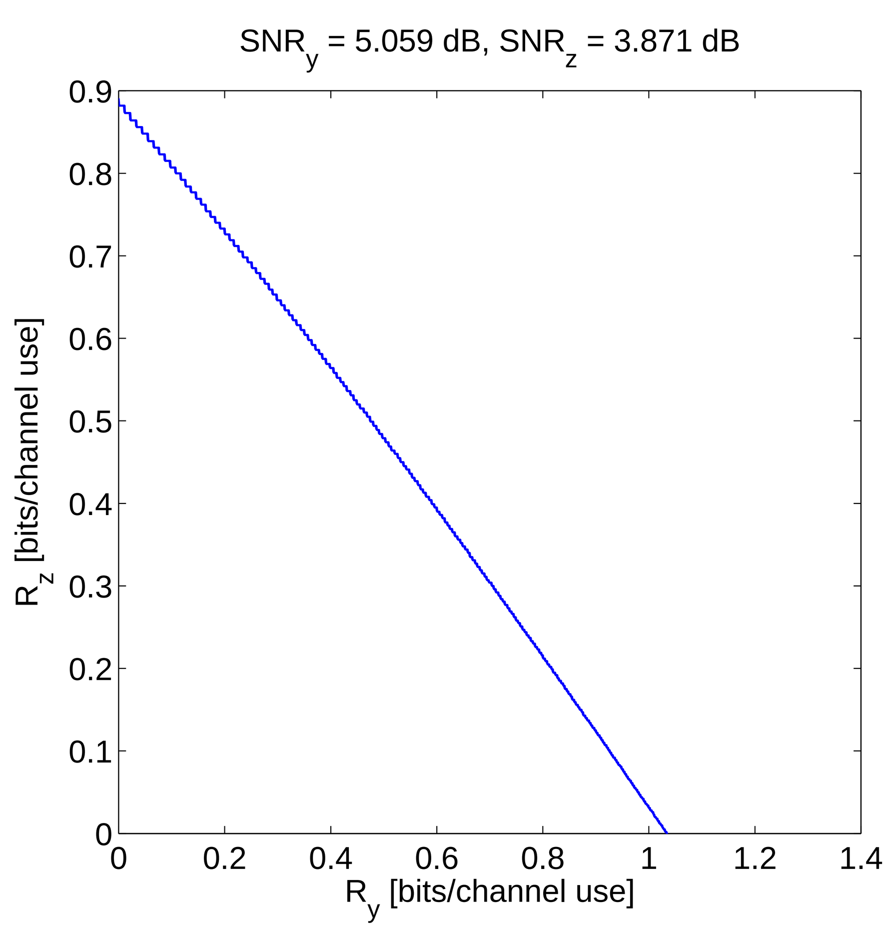
$$Y = AX + N_y, \quad Z = BX + N_z$$

- a power constraint input $\mathbb{E}(|X|^2) \leq P$

- wlog, for fixed $|A| > |B| \Longrightarrow$ degraded BC

$$\bigcup_{\alpha \in [0,1]} \left\{ \begin{array}{l} R_y \leq \frac{1}{2}\log_2 \left[1 + \alpha |A|^2 \frac{P}{N}\right] \\ R_z \leq \frac{1}{2}\log_2 \left[1 + \frac{(1-\alpha)|B|^2 P}{N + \alpha |B|^2 P}\right] \end{array} \right\}$$

- α is the fraction of input power allocated to user Y



- boundary is achieved by a Gaussian codebook

$$X = \sqrt{\alpha P} X_y + \sqrt{\bar{\alpha} P} X_z \quad (X_y, X_z) \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_2)$$

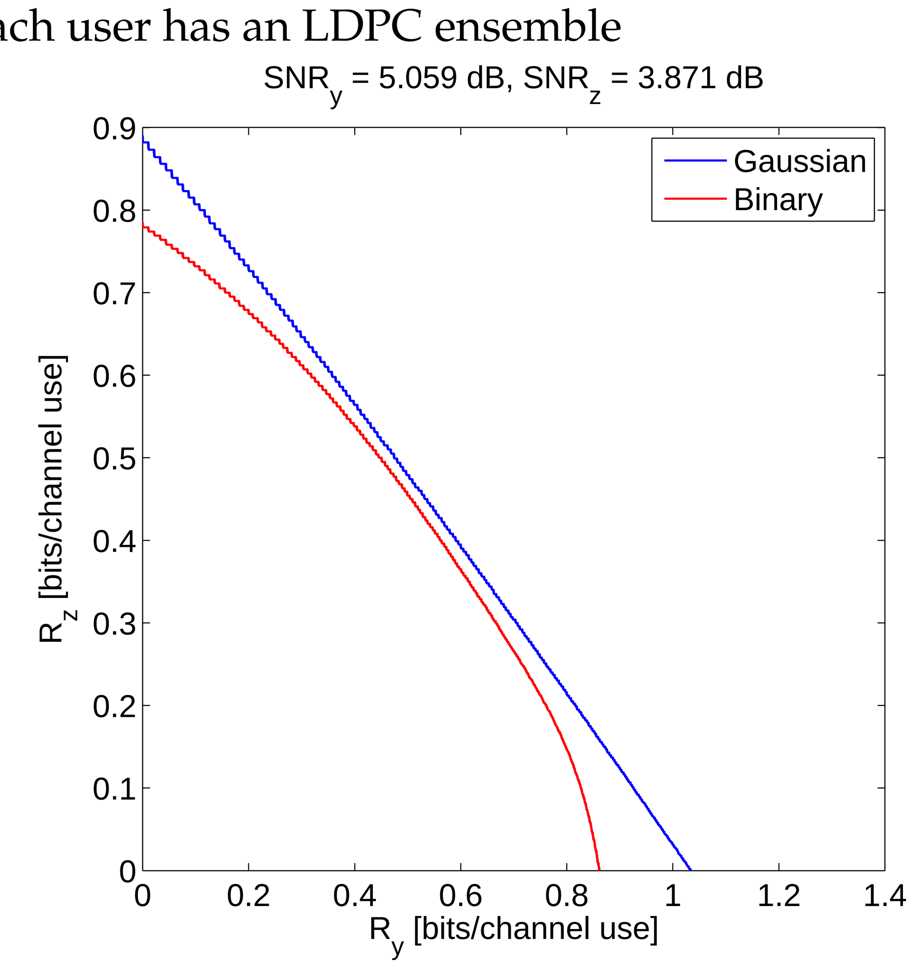
Bad news: cannot have a Gaussian codebook in practice!

Using Binary Codes

- **good news:** using a binary codebook at low SNRs, we will not lose too much! [1]
- (X_y, X_z) is uniformly picked from $\{+1, -1\}^2$

$$X = \sqrt{\alpha P} X_y + \sqrt{\bar{\alpha} P} X_z$$

- each user has an LDPC ensemble

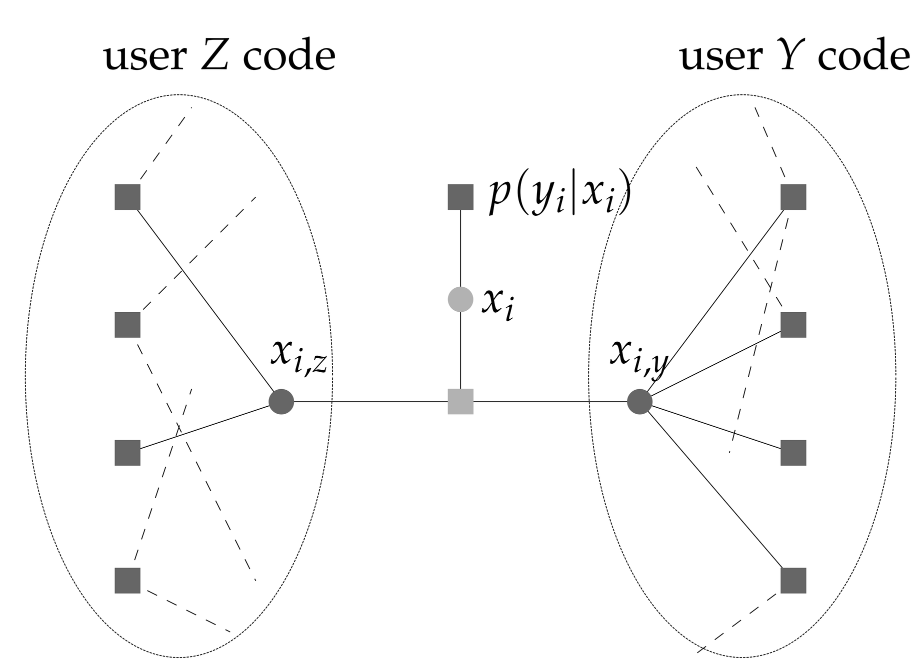


- optimal MAP detection at user Y
- mapper node handles $X = \sqrt{\alpha P} X_y + \sqrt{\bar{\alpha} P} X_z$
- mapper node is in essence an *interference canceler* [1]

Drawbacks

- joint decoding at both users
- both codes are required at the receivers

Question What if we could get rid of the mapper node?

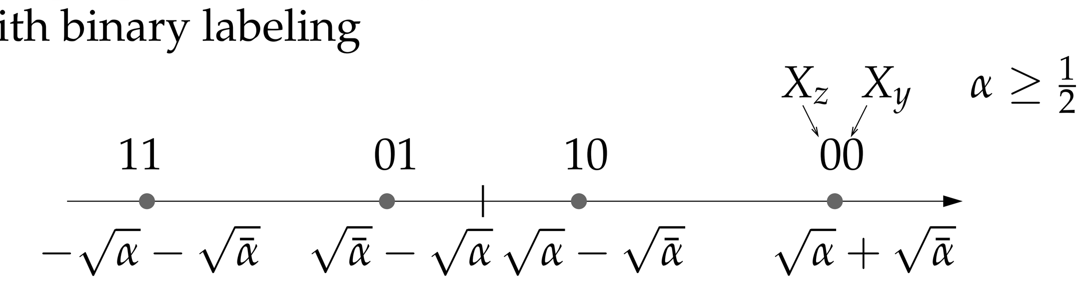


Bit-Interleaved Coded Modulation (BICM)

- **Caire [2]** BICM performs extremely close to the optimal decoder with a lower complexity if Gray labeling is used.
- using Gray labeling, there is a minor dependency among label bits

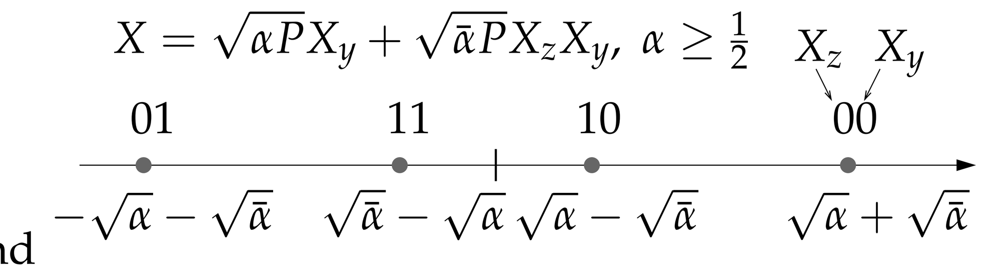
Proposed Method

- superposition coding is a 4-PAM-like modulation with binary labeling



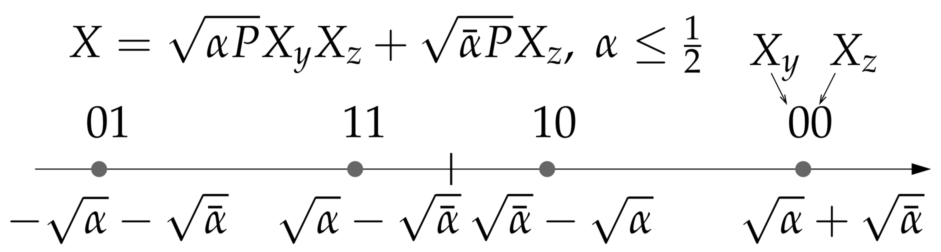
- LDPC codes are self-interleaved
- can apply Gray labeling

$$X = \sqrt{\alpha P} X_y + \sqrt{\bar{\alpha} P} X_z X_y, \ \alpha \geq \frac{1}{2}$$

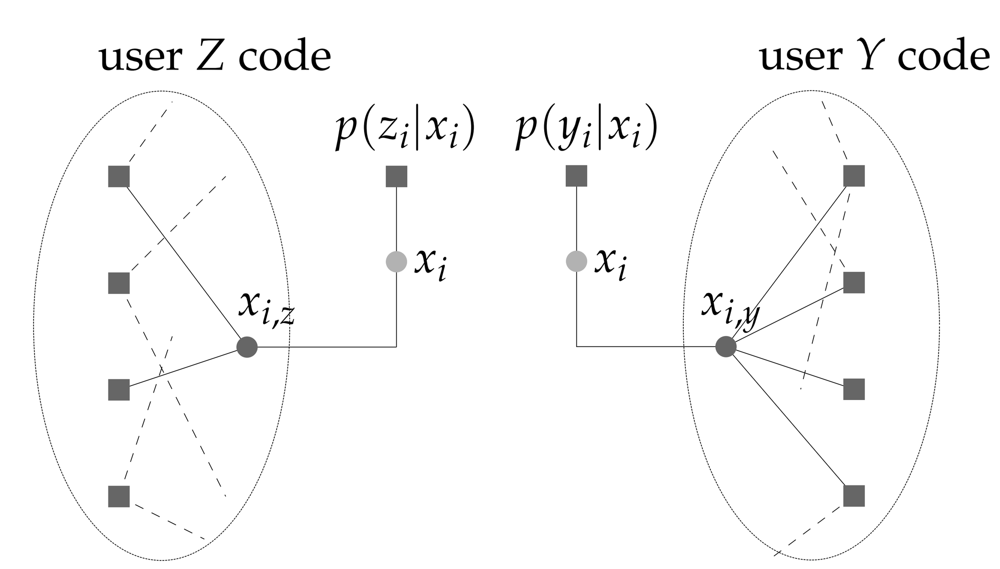


and

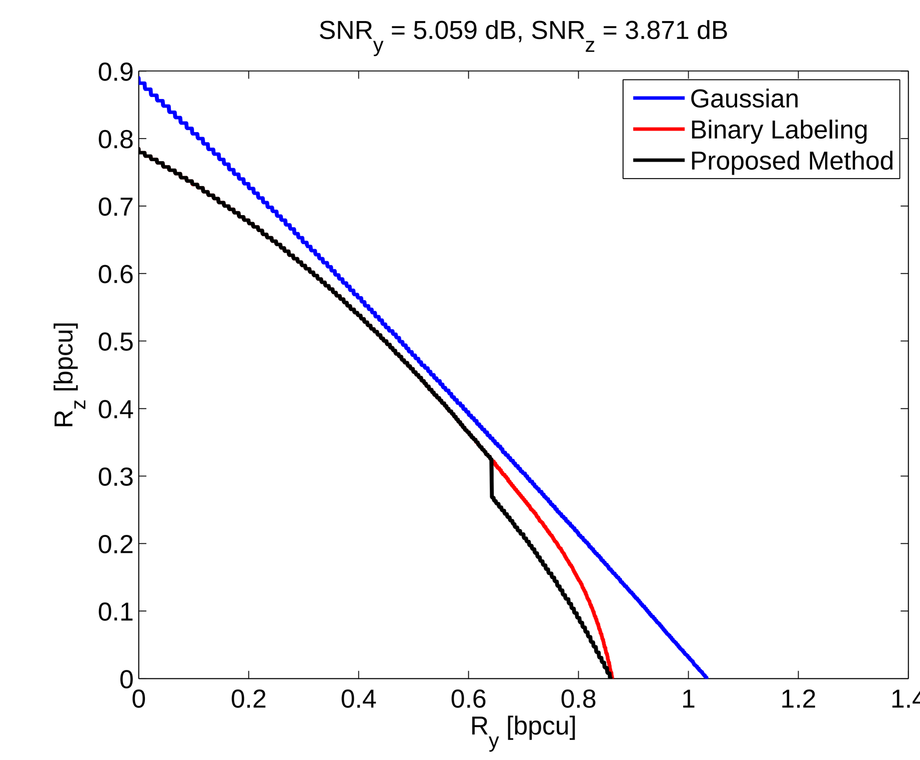
$$X = \sqrt{\alpha P} X_y X_z + \sqrt{\bar{\alpha} P} X_z, \ \alpha \leq \frac{1}{2}$$



- trying to reduce the dependency of code bits
- does not exactly match superposition coding
- may get a non-convex region due to nonlinearity
- due to minor dependency, we propose to remove mapper nodes



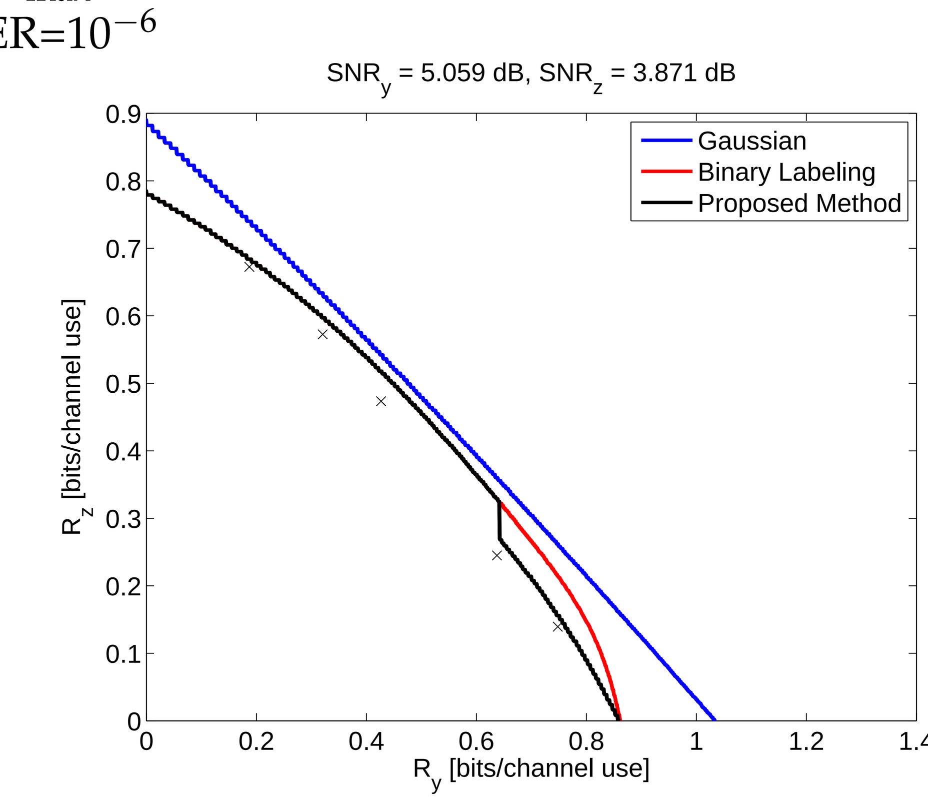
- each user only needs to have its own code
- no need for joint decoding
- compared to the optimal method, decoding complexity is decreased at least by a factor of 50%



- most of the region is covered by the proposed method
- as expected, the region is not convex since we adaptively force labeling to be Gray!

Simulation Results

- can optimize a pair of LDPC codes to get close to the obtained region
- $\mathrm{LLR}_{\max} = 25$ and a 9-bit quantizer
- $\lambda_{\max} = 50$ with at most 800 iterations to a target BER$=10^{-6}$



References

[1] P. Berlin and D. Tuninetti, "LDPC codes for fading gaussian broadcast channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 6, pp. 2173–2182, Jun. 2005.

[2] G. Caire, G. Taricco, and E. Biglieri, "Bit-interleaved coded modulation," *IEEE Trans. Inf. Theory*, vol. 44, no. 3, pp. 927–946, May 1998.