Feedback in Wireless Networks

Recent Results and Discoveries

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Joint work with Ravi Tandon, et al.

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Outline

- Background
  - Point-to-point channels
  - Multi-terminal channels
- Static Interference Channels
  - Why feedback helps
  - Feedback gain for many-user interference channels
- Fading MISO Broadcast Channels
  - The effects of channel state feedback
  - Spatio-temporal variation in channel state feedback

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Background: Point-to-Point Channels

- Infinite blocklengths:
  - Feedback does not increase capacity (Shannon, IT'56)
  - But, feedback can speed-up the convergence of the error probability to zero (Schalkwijk-Kailath, IT'66)
- Finite blocklengths:
  - Feedback can dramatically improve the maximal achievable rate (Polyanskiy-Poor-Verdu, IT'11)

Background: Multi-terminal Channels

- Feedback does increase capacity; e.g. (among many others):
  - Multiple-access channels (Gaarder-Wolf, IT'75)
  - Broadcast channels (Ozarow & Leung-Yan-Cheong, IT'84)
  - Wiretap channels (Leung-Yan-Cheong, PhD Thesis'76)
  - Relay channels (Willems-Van der Meulen, IT'83)

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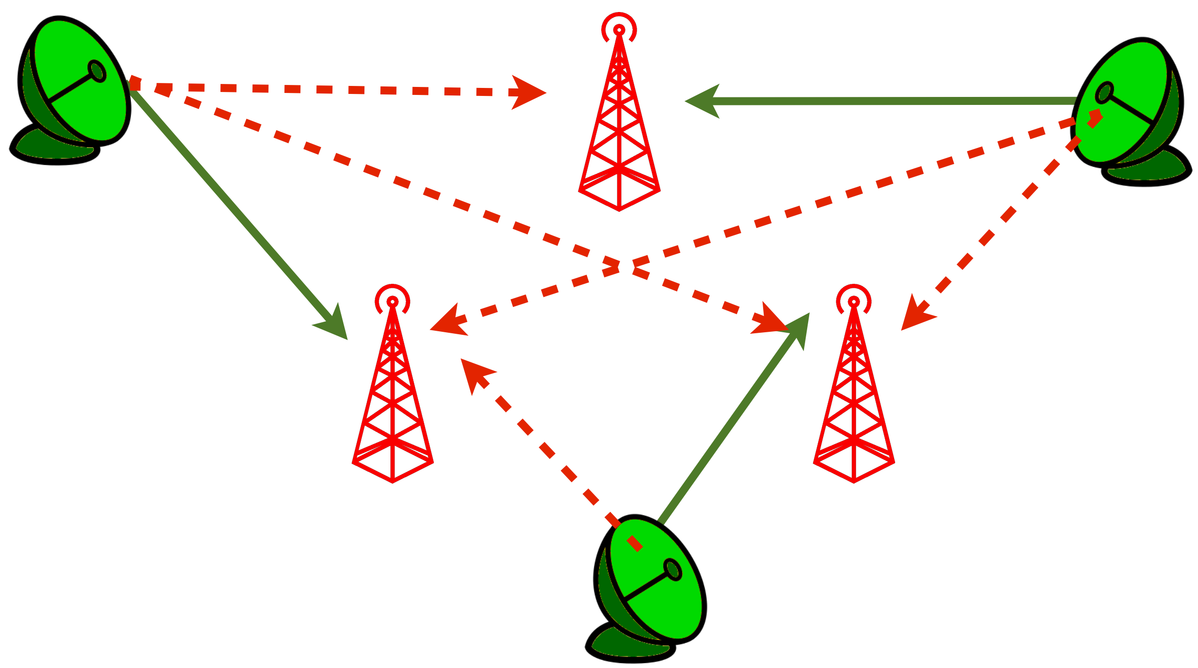
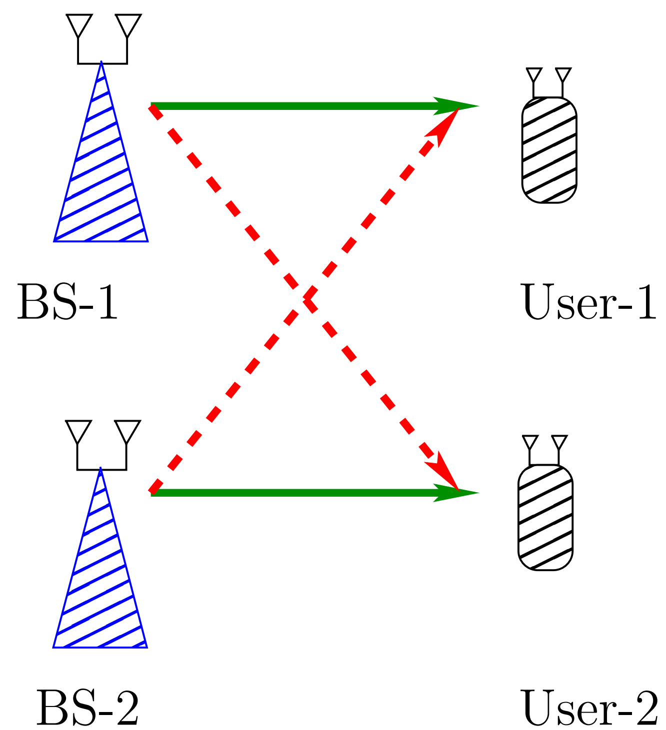
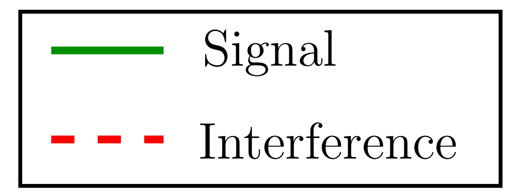
Interference in Wireless Networks



- Broadcast nature of wireless medium
- Spectrum reuse → interference is unavoidable
- Fundamental barrier to spectral efficiency

Two-User Gaussian Interference Channel





$$Y_1 = \sqrt{\mathbf{SNR}}X_1 + \sqrt{\mathbf{INR}}X_2 + Z_1$$
$$Y_2 = \sqrt{\mathbf{INR}}X_1 + \sqrt{\mathbf{SNR}}X_2 + Z_2$$

- Canonical model for interfering users
- Static setting: SNR, INR fixed throughout communication
- Capacity region is unknown

Degrees of Freedom

Point-to-Point AWGN Channel

$$Y = \sqrt{\mathrm{SNR}} X + N \qquad \mathbb{E}[X^2] \leq 1, \quad N \sim \mathcal{N}(0,1)$$

$$C = \tfrac{1}{2}\log(1 + \mathrm{SNR})$$

$$\begin{aligned} \mathrm{DoF} &= \lim_{\mathrm{SNR}\to\infty} \frac{C}{\frac{1}{2}\log(\mathrm{SNR})} \\ &= 1 \end{aligned}$$

DoF is a measure of how capacity scales with SNR.

Generalized Degrees of Freedom

Normalization (per-user)

Sum-rate

$$\mathbf{GDoF}(\alpha) = \frac{1}{2} \limsup_{\mathbf{SNR}\to\infty} \frac{\max\left(R_1 + R_2\right)}{\frac{1}{2}\log(\mathbf{SNR})}$$

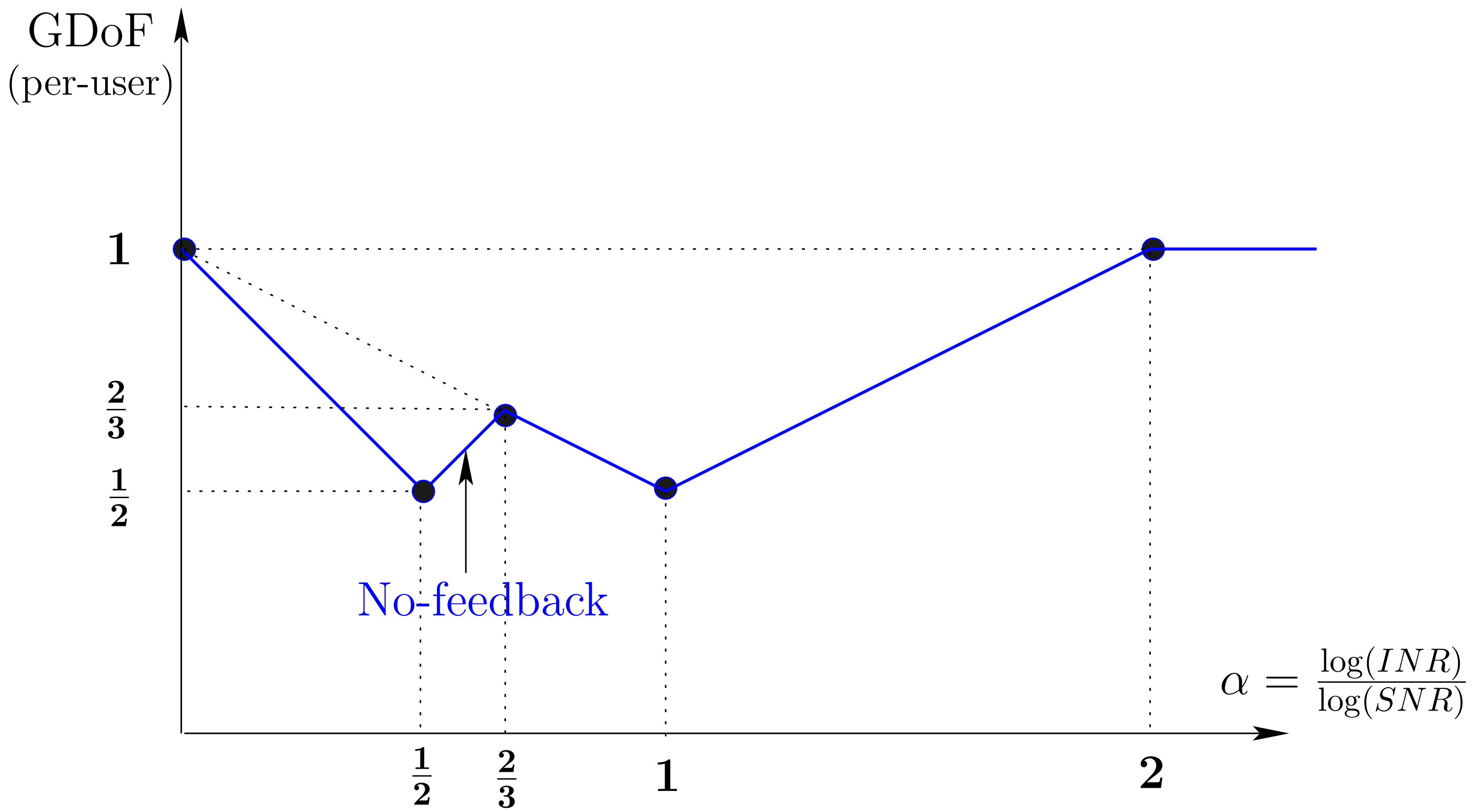
Interference free rate

$$\alpha = \frac{\log(\mathbf{INR})}{\log(\mathbf{SNR})}$$

Interference parameter

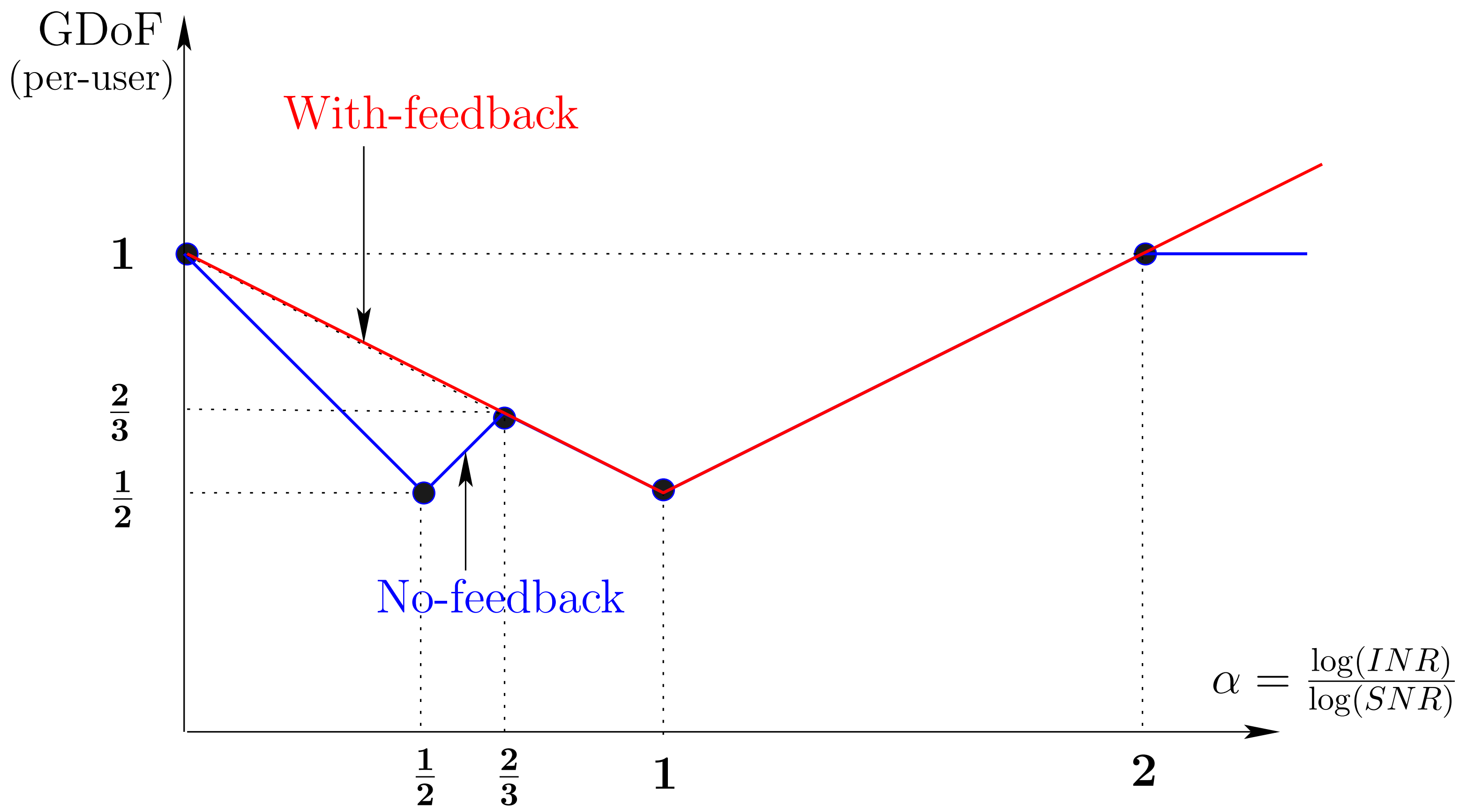
- GDoF captures behavior when SNR, INR are high
- System is constrained by interference (not by noise)

GDoF without Feedback



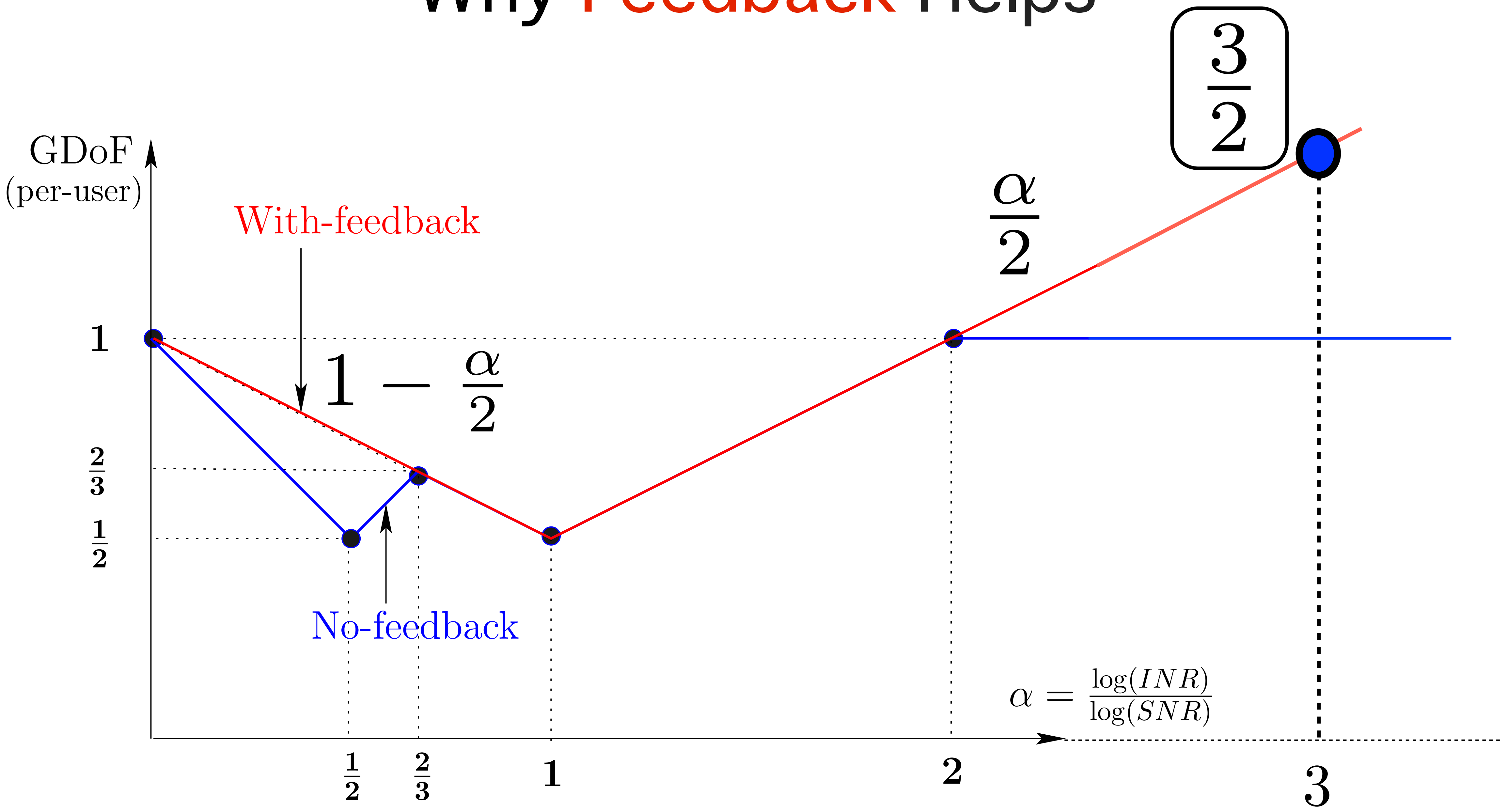
- GDoF is a W-curve [Etkin-Tse-Wang IT'08]
- Saturates beyond 2 [very-high interference]

GDoF with Feedback

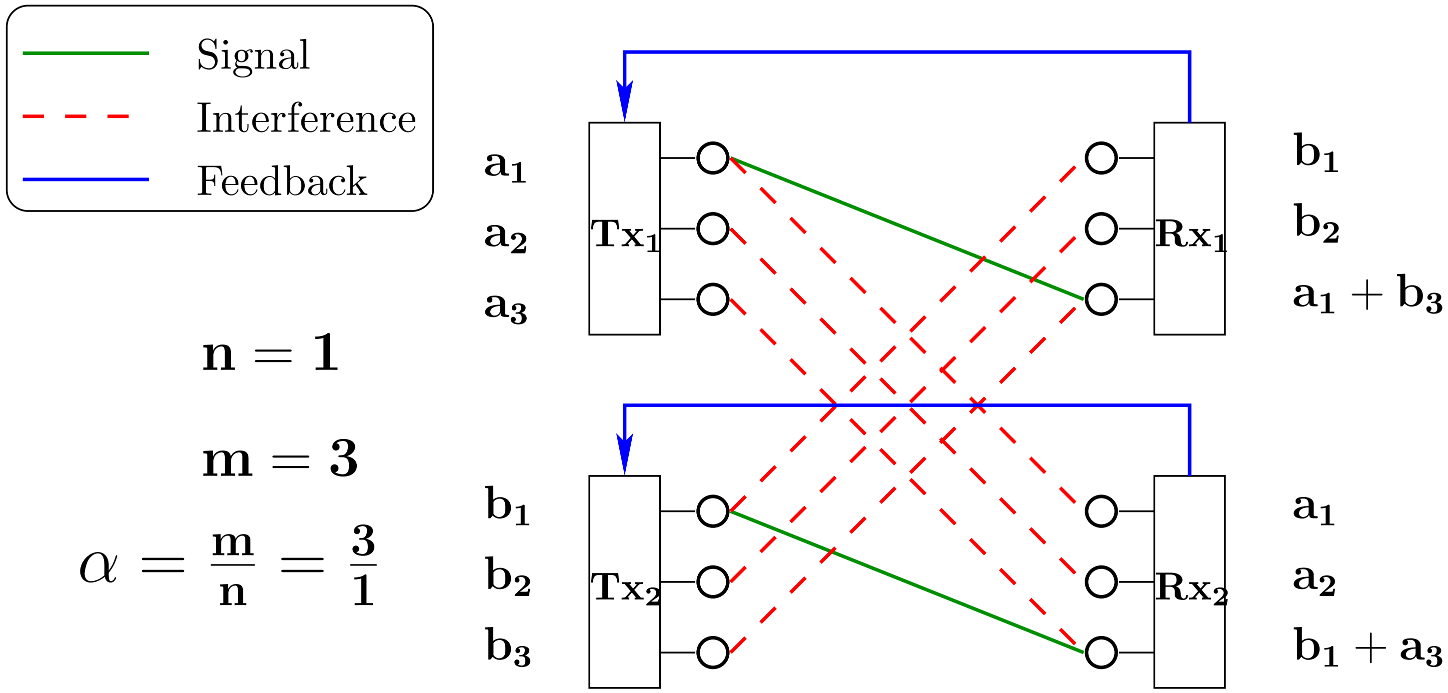


- GDoF is a V-curve [Suh-Tse, IT'11]
- Increasing beyond 2 [very-high interference].

Why Feedback Helps



Intuition Via Linear Deterministic Model



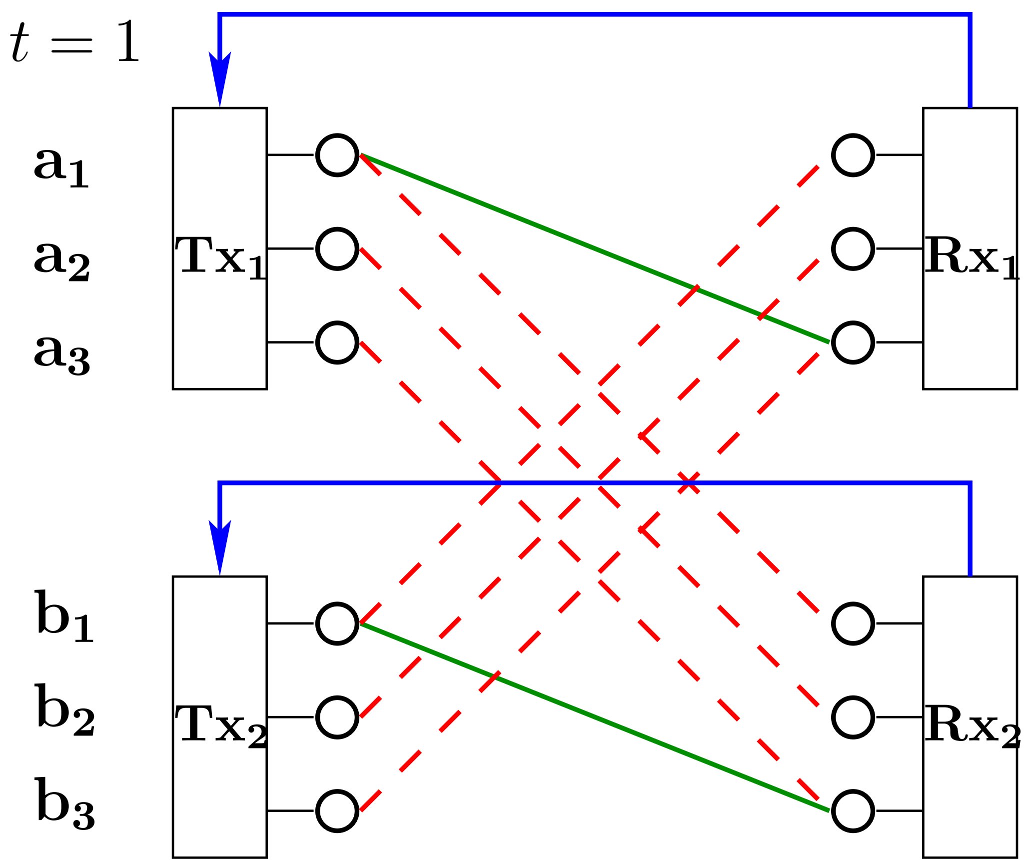
$$n = \lfloor \log(\mathrm{SNR}) \rfloor$$
$$m = \lfloor \log(\mathrm{INR}) \rfloor$$

- Linear Deterministic Interference Channel

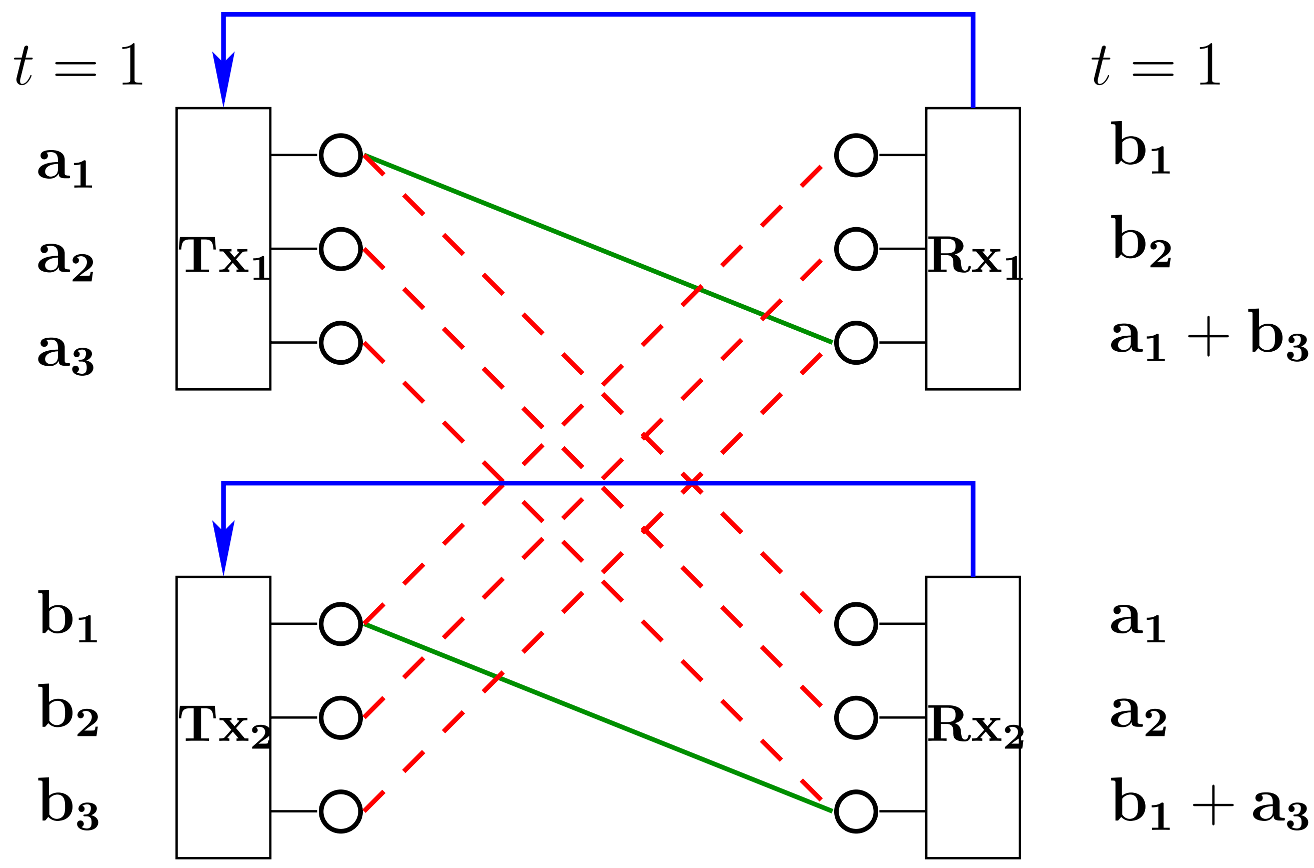
$$y_1 = \lfloor 2^n x_1 \rfloor \oplus \lfloor 2^m x_2 \rfloor$$
$$y_2 = \lfloor 2^m x_1 \rfloor \oplus \lfloor 2^n x_2 \rfloor$$

- Approximation for Gaussian Interference Channel

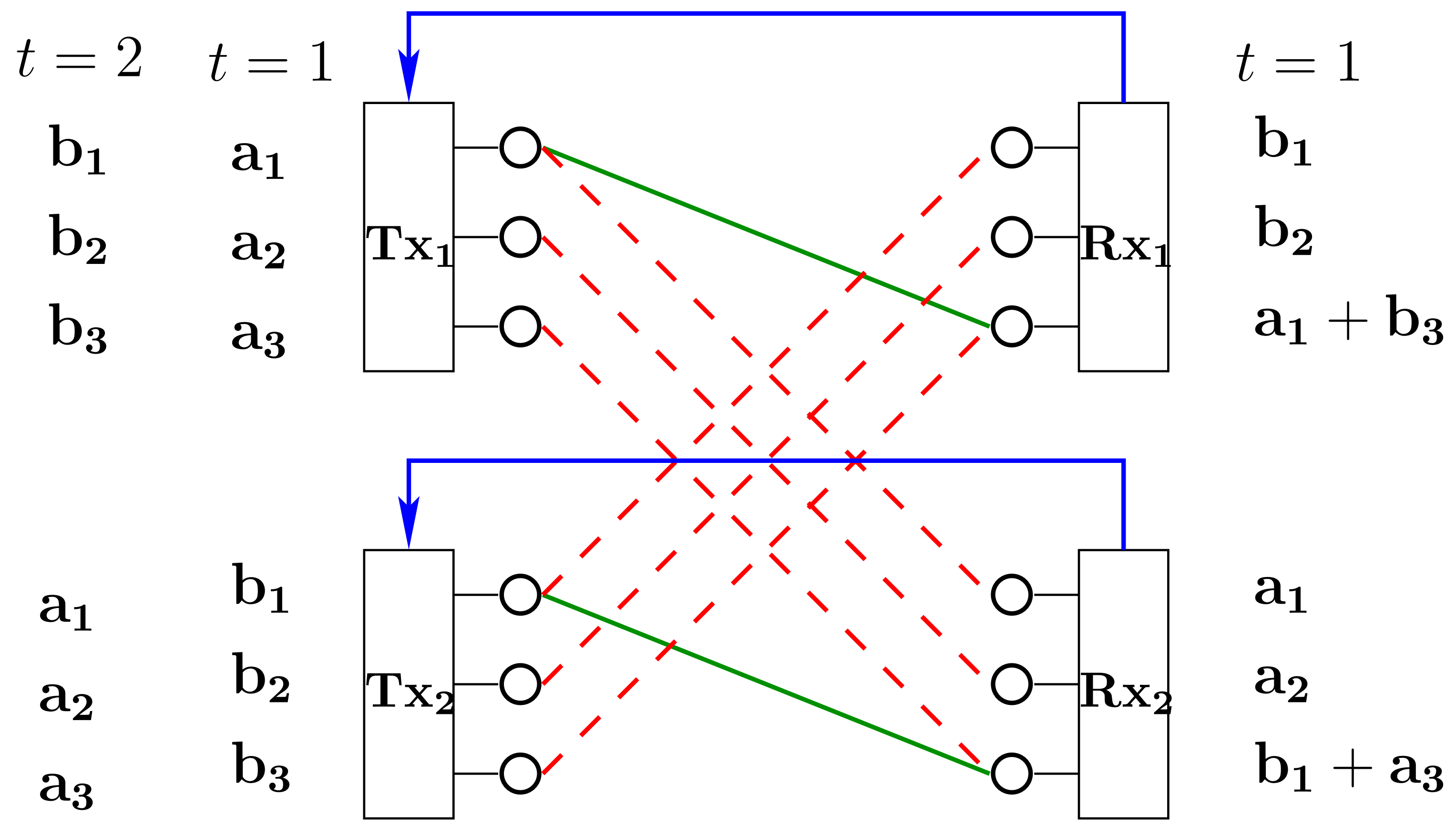
Achieving 3/2 (per-user) with Feedback



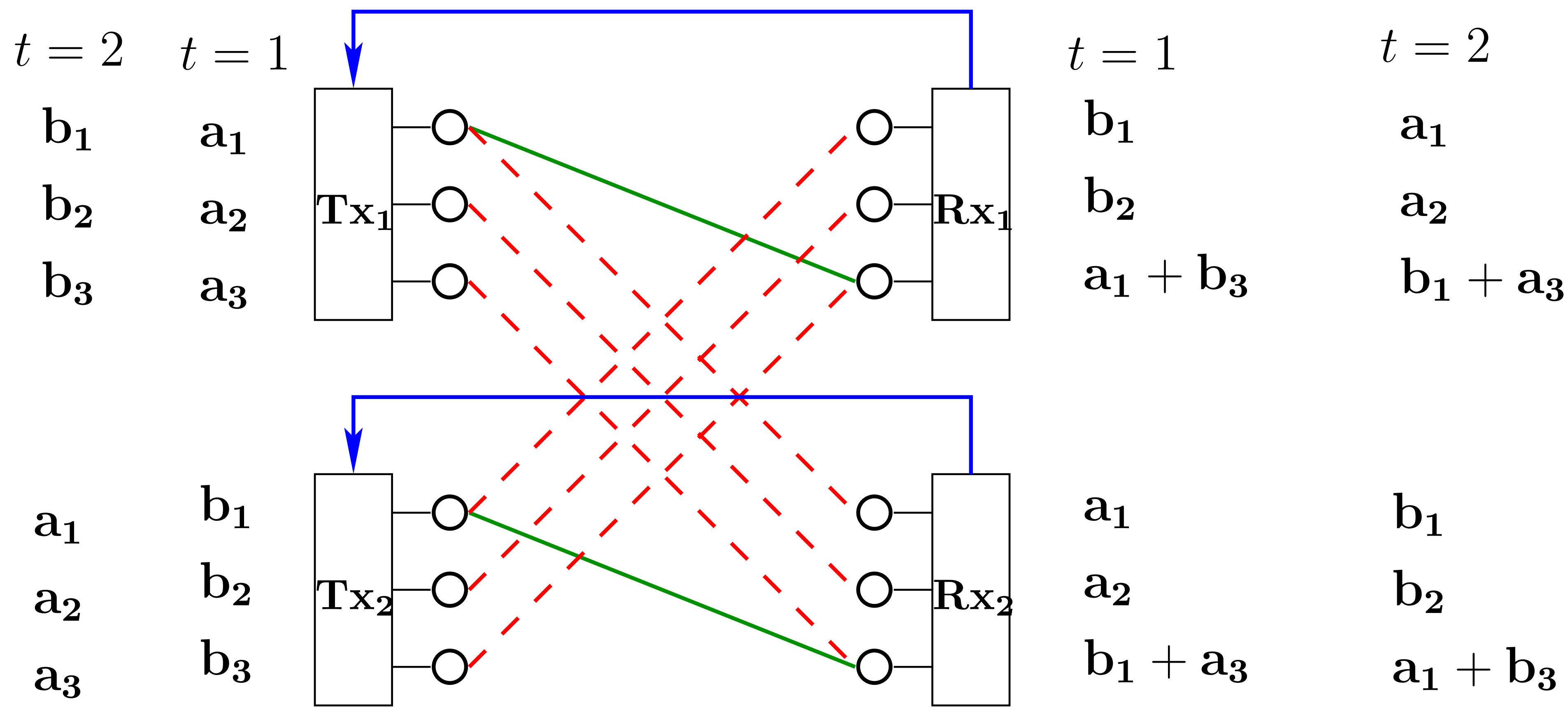
Achieving 3/2 (per-user) with Feedback



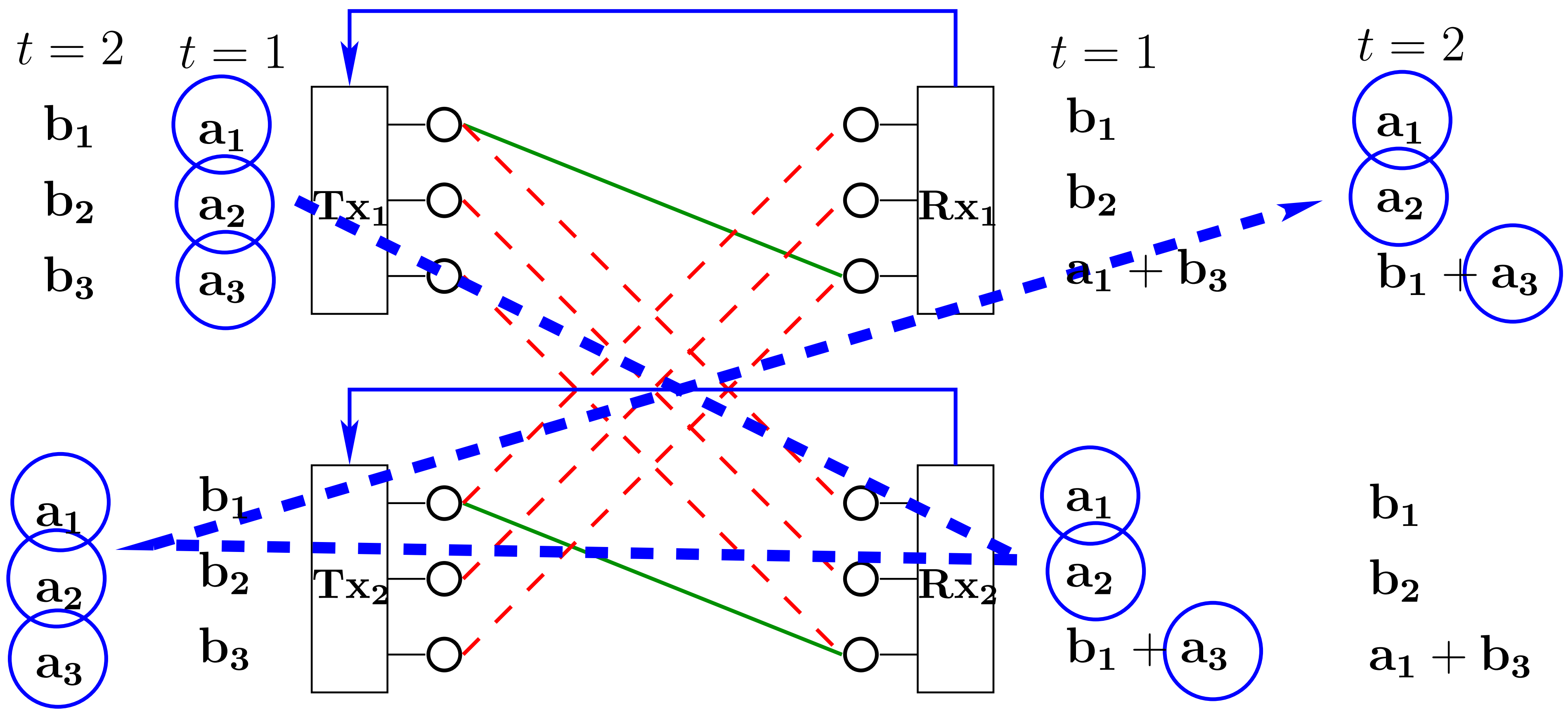
Achieving 3/2 (per-user) with Feedback



Achieving 3/2 (per-user) with Feedback



Feedback Provides Alternative Path to Rx



Natural Questions

Q1: Do these results extend to more than two users?

Q2: If yes, how much does feedback help?

Q3: Dependence of feedback gains on network topology?

Natural Questions

Q1: Do these results extend to more than two users?

A1: Yes, to (at least) fully connected and ring networks.

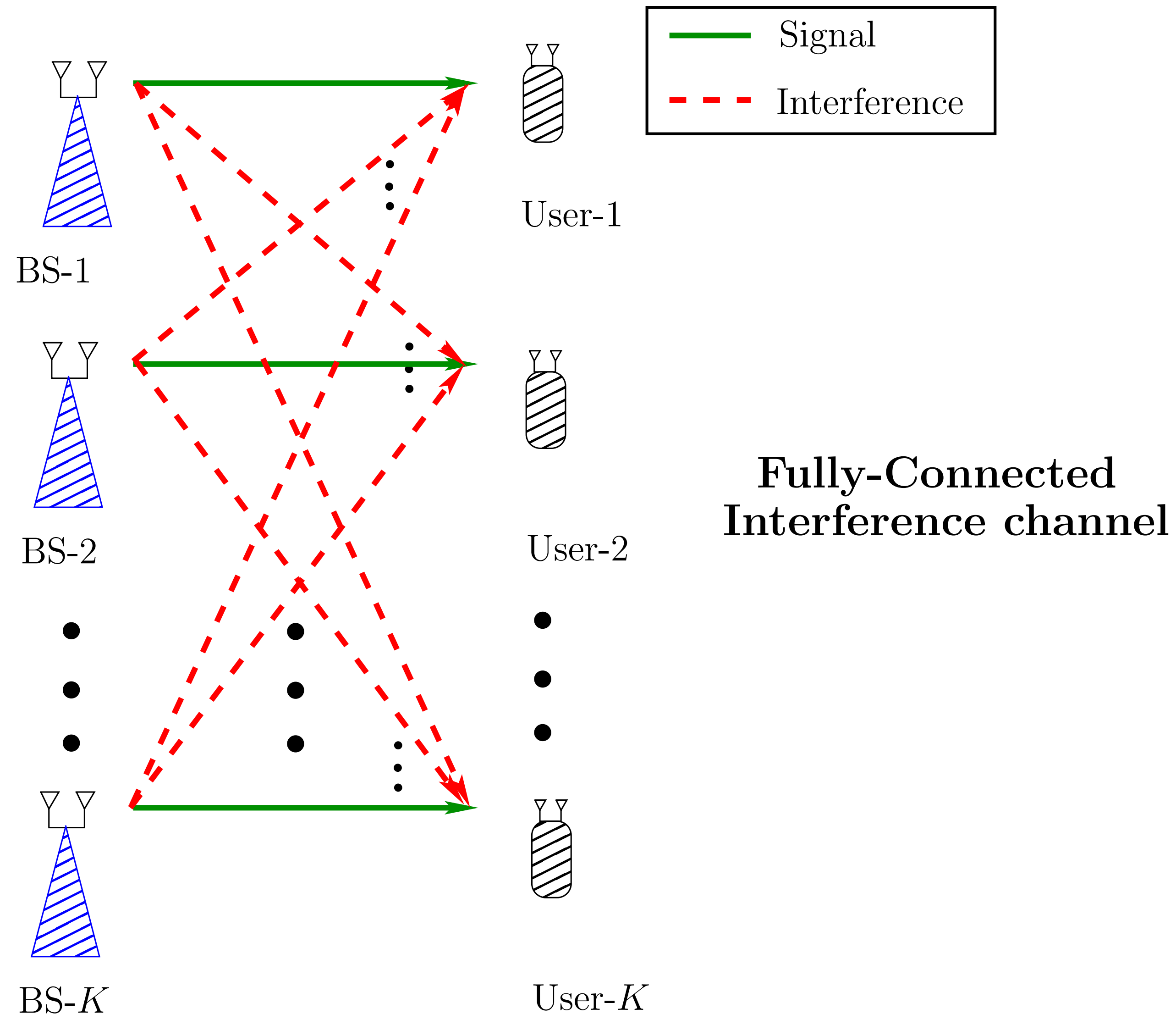
Q2: If yes, how much does feedback help?

A2: Sometimes, feedback provides unbounded gains.

Q3: Dependence of feedback gains on network topology?

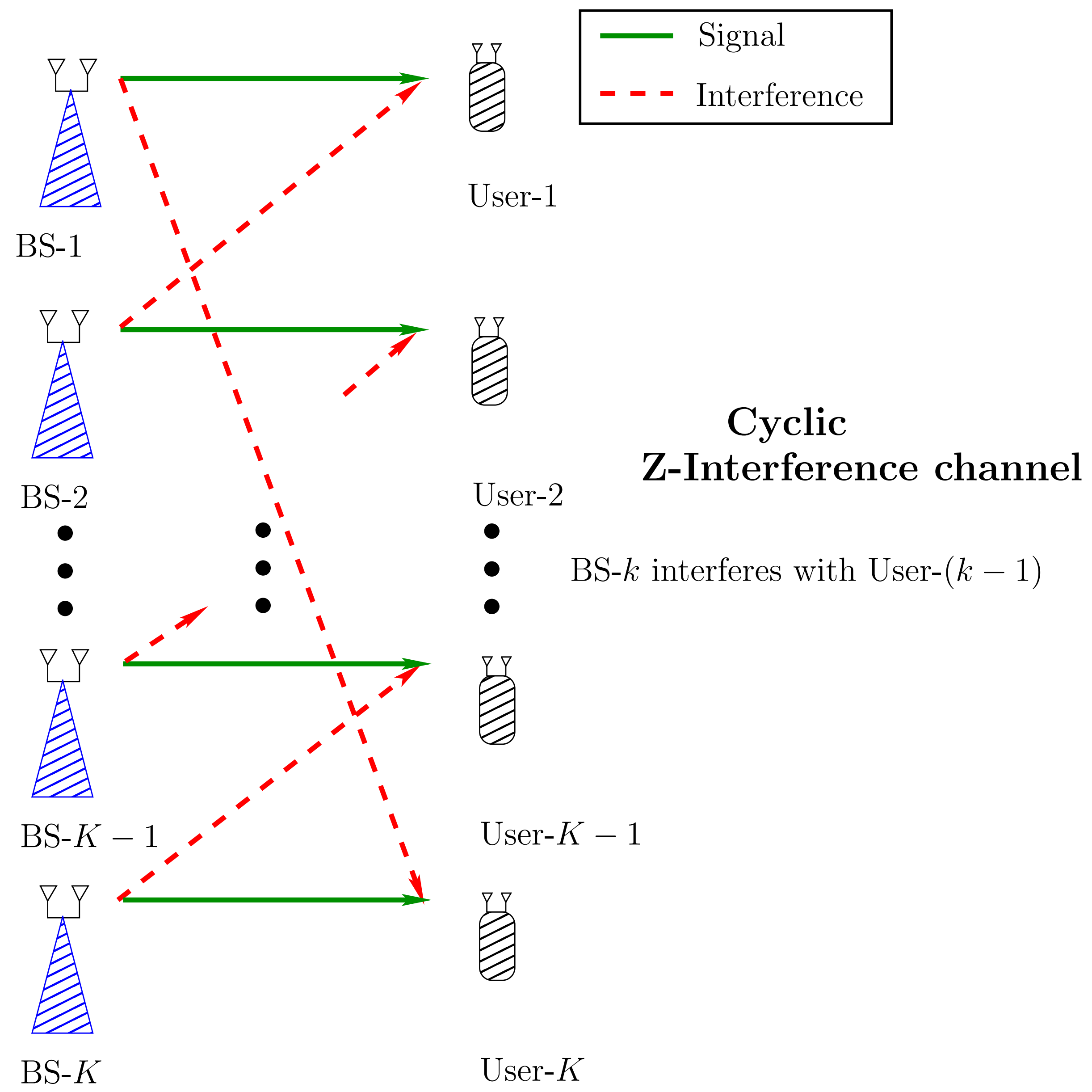
A3: In general, feedback gain depends on topology.

Fully Connected K-user Interference Channel



- Natural generalization of 2-user IC
- Every base-station interferes with every user

Cyclic K-user Interference Channel



- Inspired by Wyner model for cellular network
- BS k interferes with user (k-1)

Known Results: GDoF without Feedback

Fully Connected IC [Jafar-Viswanath, IT'10]

$$GDoF_{FC}^{No-FB}(\alpha) = \begin{cases} 1-\alpha, & \alpha \in [0, 1/2) \\ \alpha, & \alpha \in [1/2, 2/3) \\ \alpha/2, & \alpha \in [2/3, 1) \\ 1/K, & \alpha = 1 \\ 1-\alpha/2, & \alpha \in (1, 2) \\ 1, & \alpha > 2. \end{cases}$$

Cyclic IC [Zhou-Yu, IT'13]

$$GDoF_{Cyclic}^{No-FB}(\alpha) = \begin{cases} 1-\alpha, & \alpha \in [0, 1/2) \\ \alpha, & \alpha \in [1/2, 2/3) \\ \alpha/2, & \alpha \in [2/3, 1) \\ 1-\alpha/2, & \alpha \in [1, 2) \\ 1, & \alpha \geq 2. \end{cases}$$

Our Contribution: GDoF with Feedback

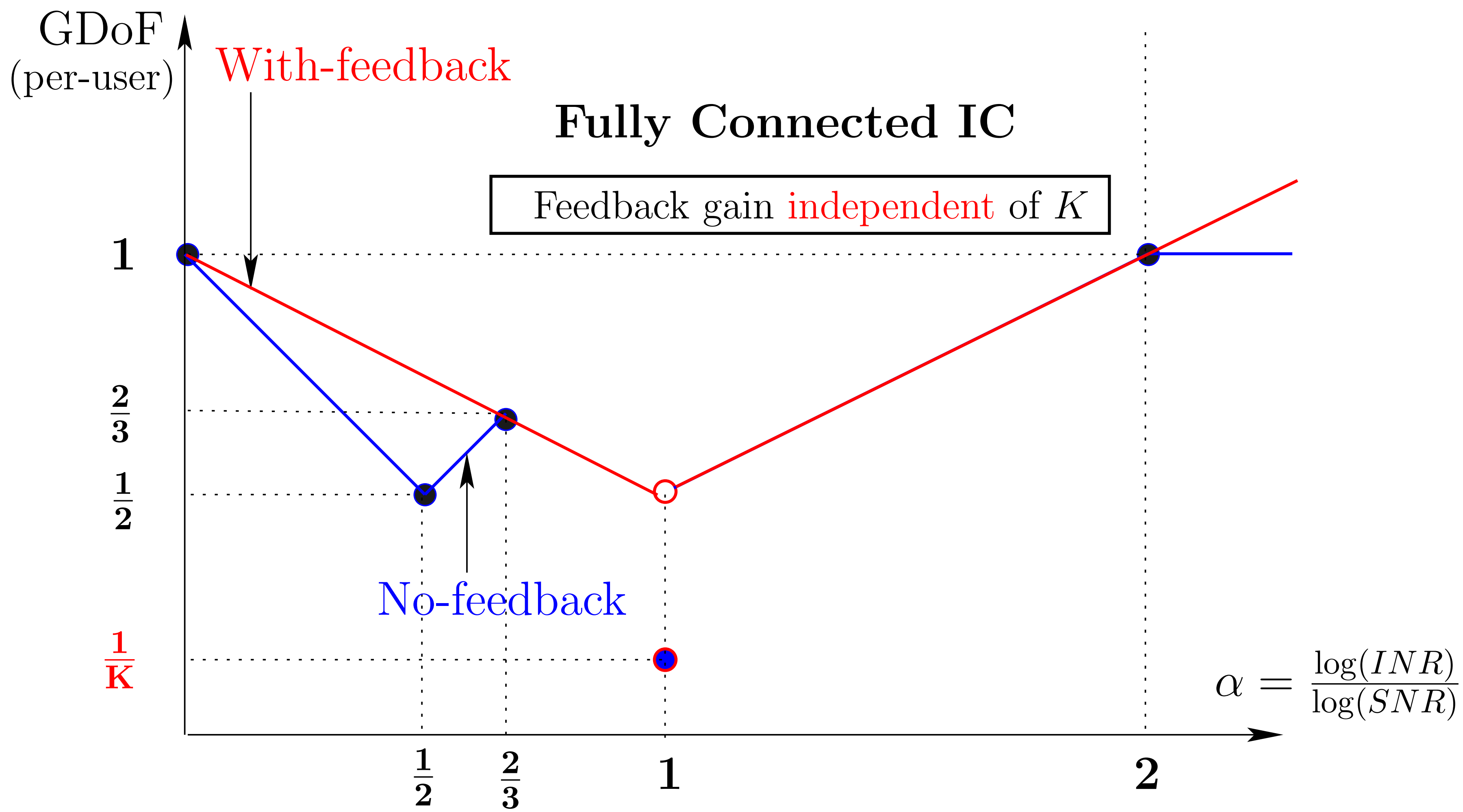
Fully Connected IC [Mohajer-Tandon-Poor IT'13]

$$GDoF^{FB}_{FC}(\alpha) = \begin{cases} 1 - \alpha/2, & \alpha \in [0,1) \\ 1/K, & \alpha = 1 \\ \alpha/2, & \alpha \in (1,\infty). \end{cases}$$

Cyclic IC [Tandon-Mohajer-Poor IT'13]

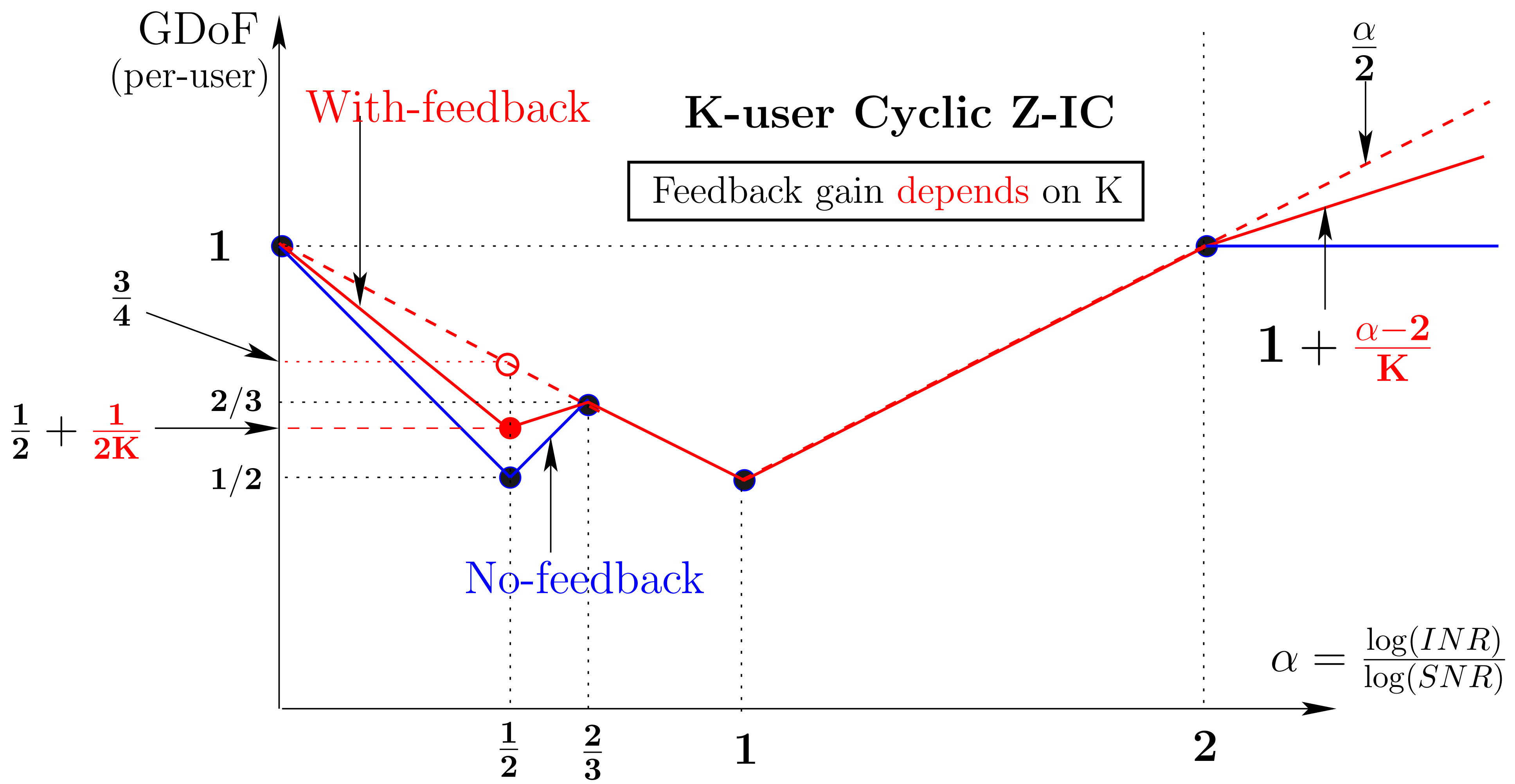
$$GDoF^{FB}_{Cyclic}(\alpha) = \begin{cases} 1 - \alpha + \frac{\alpha}{K}, & \alpha \in [0,1/2) \\ \alpha + \frac{2-3\alpha}{K}, & \alpha \in [1/2,2/3) \\ \alpha/2, & \alpha \in [2/3,1) \\ 1 - \alpha/2, & \alpha \in [1,2) \\ 1 + \frac{\alpha-2}{K}, & \alpha \geq 2. \end{cases}$$

GDoF Curves with and without Feedback



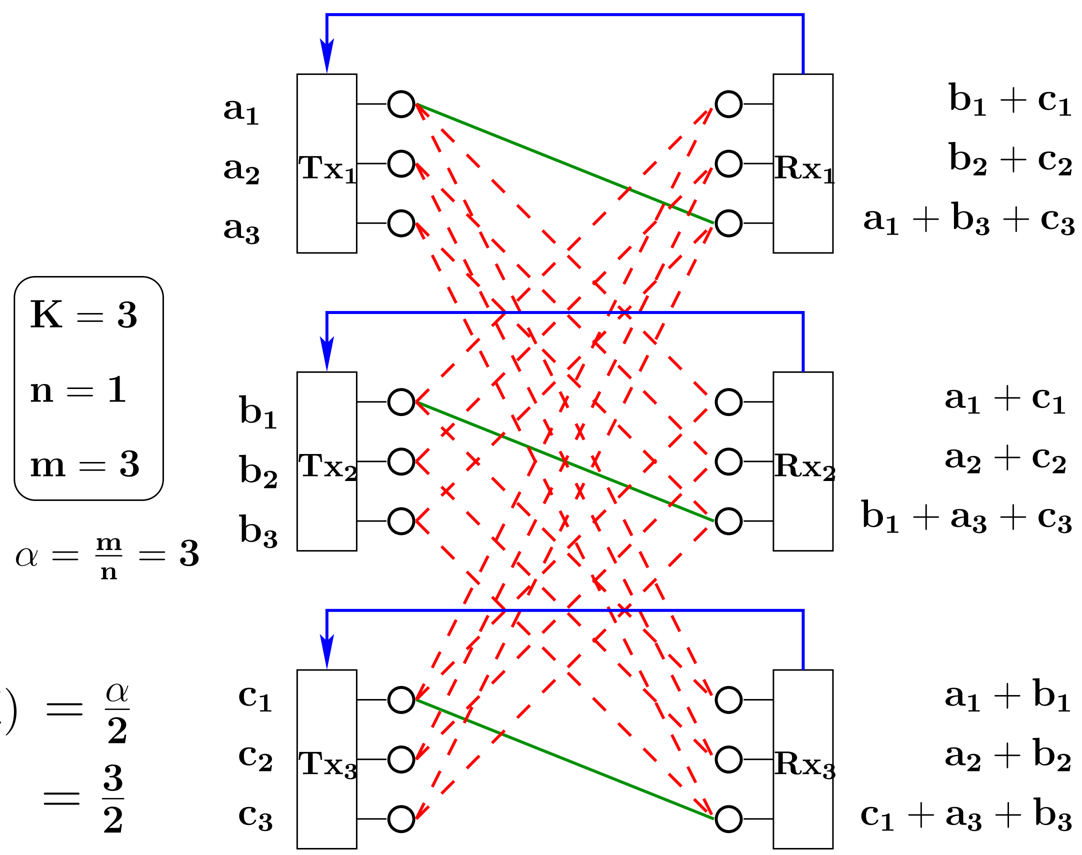
Per-user feedback gain is independent of K.

GDoF Curves with and without Feedback



Per-user feedback gain depends on K.
As K increases, V-curve ----> W-Curve

3-user Fully Connected Interference Channel

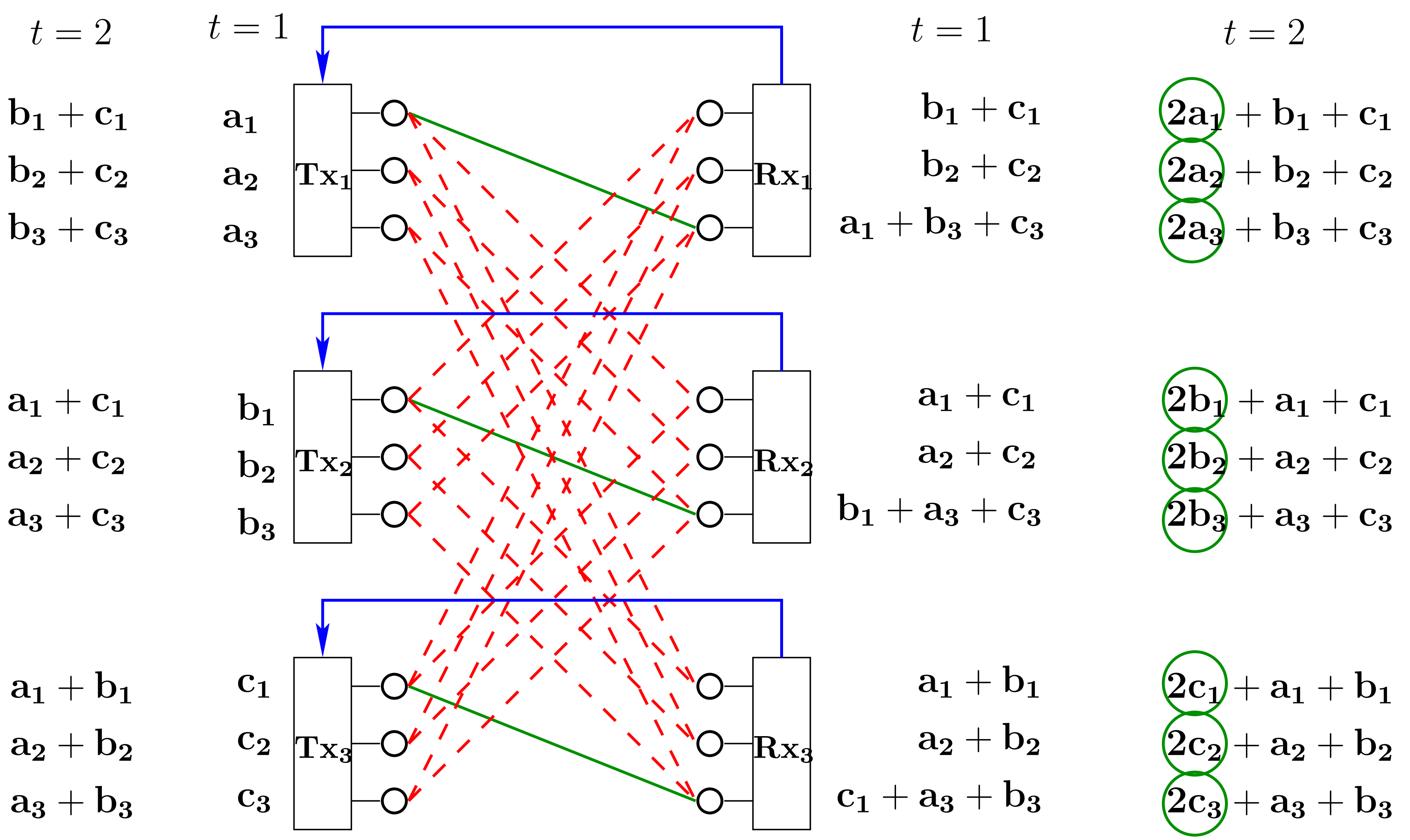


$$\mathbf{GDoF}(\alpha, \mathbf{K}) = \frac{\alpha}{2} = \frac{3}{2}$$

(per-user)

Can feedback help in transmission of 3 bits per user in 2 channel uses ?

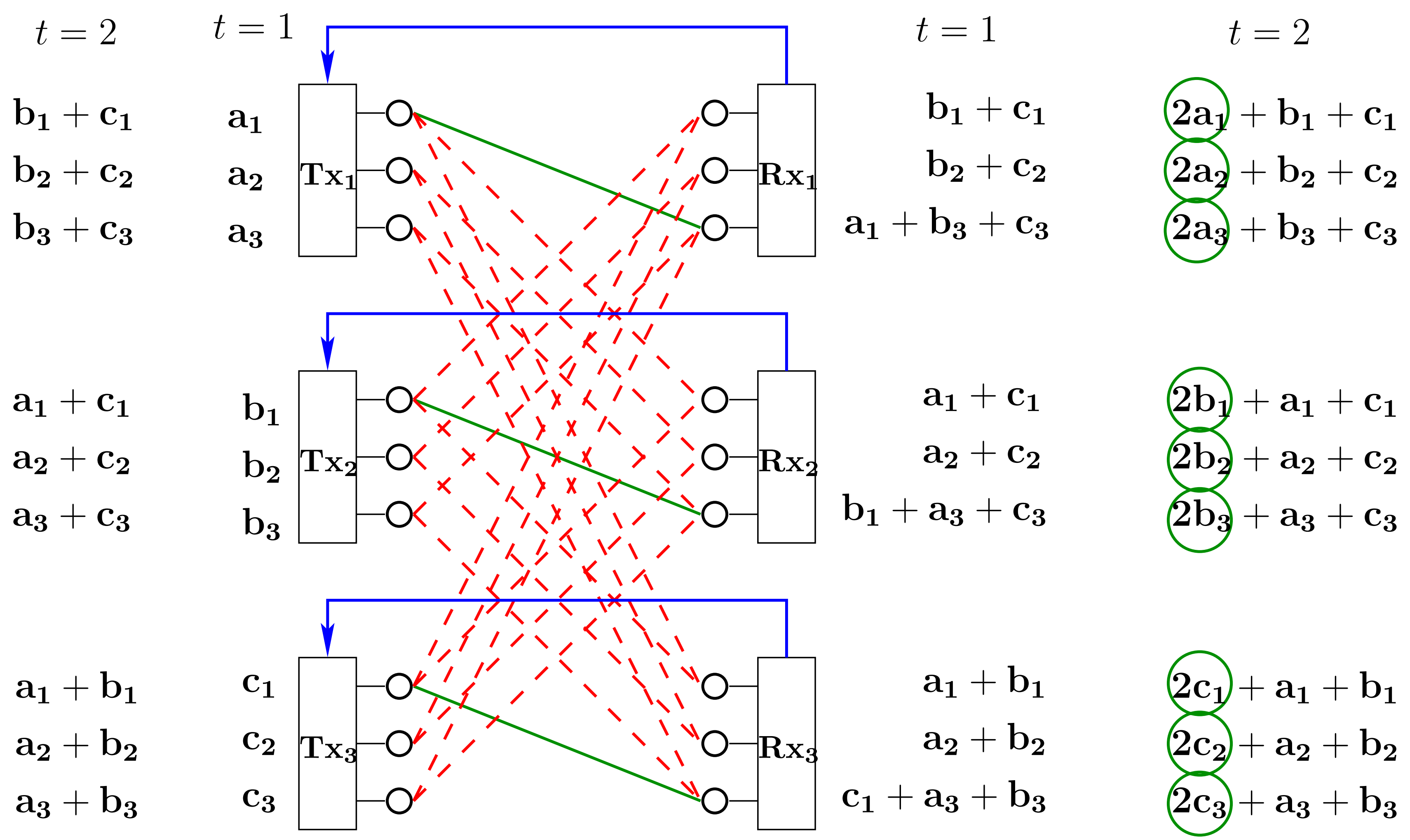
Coding Scheme: Main Idea



Transmitters decode net-interference viafFeedback

Interference at t=2 should be the same as the clean signal at t=1.

Translation to the Gaussian Model



Sum of two-(or more)-codewords should be a codeword.

Nested Lattice Codes for interference alignment.

Decoding of lattice codeword(s) ⟶ cancel off to decode signal.

Summary: Static Interference Channels

- Feedback can help exploit alternative paths to the receivers
- Significant capacity gains possible
- Connections of feedback gains to network topology
- More interference does not necessarily imply less feedback gain

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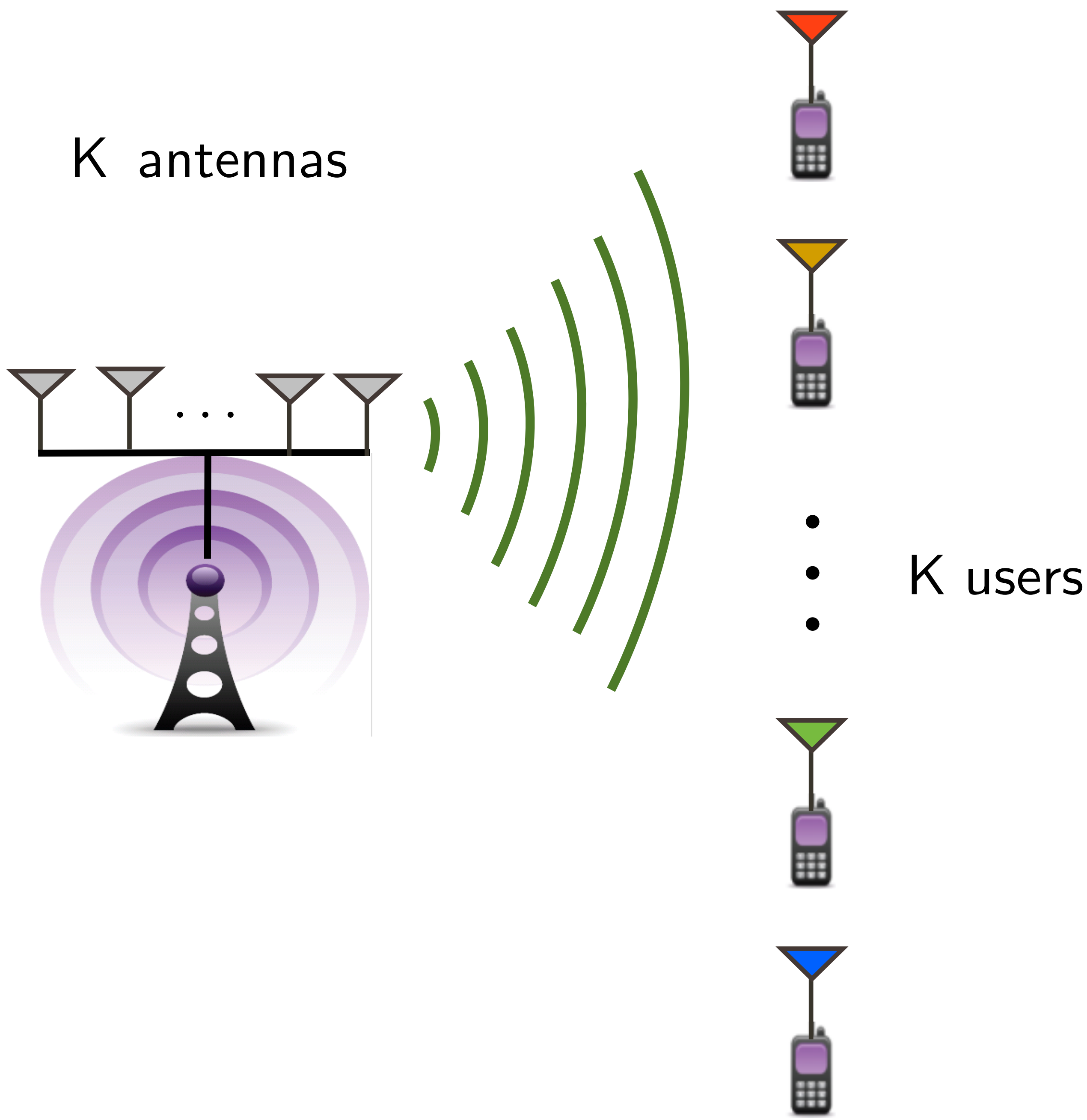
Interference Mitigation via MIMO

- Downlink multi-user MIMO (spatial multiplexing)
- Inter-cell interference mitigation
- Coordinated multi-point (CoMP in LTE)
- Key enabler in all approaches:

- Accurate & timely channel knowledge at transmitter(s)

Focus: K-user Downlink MISO



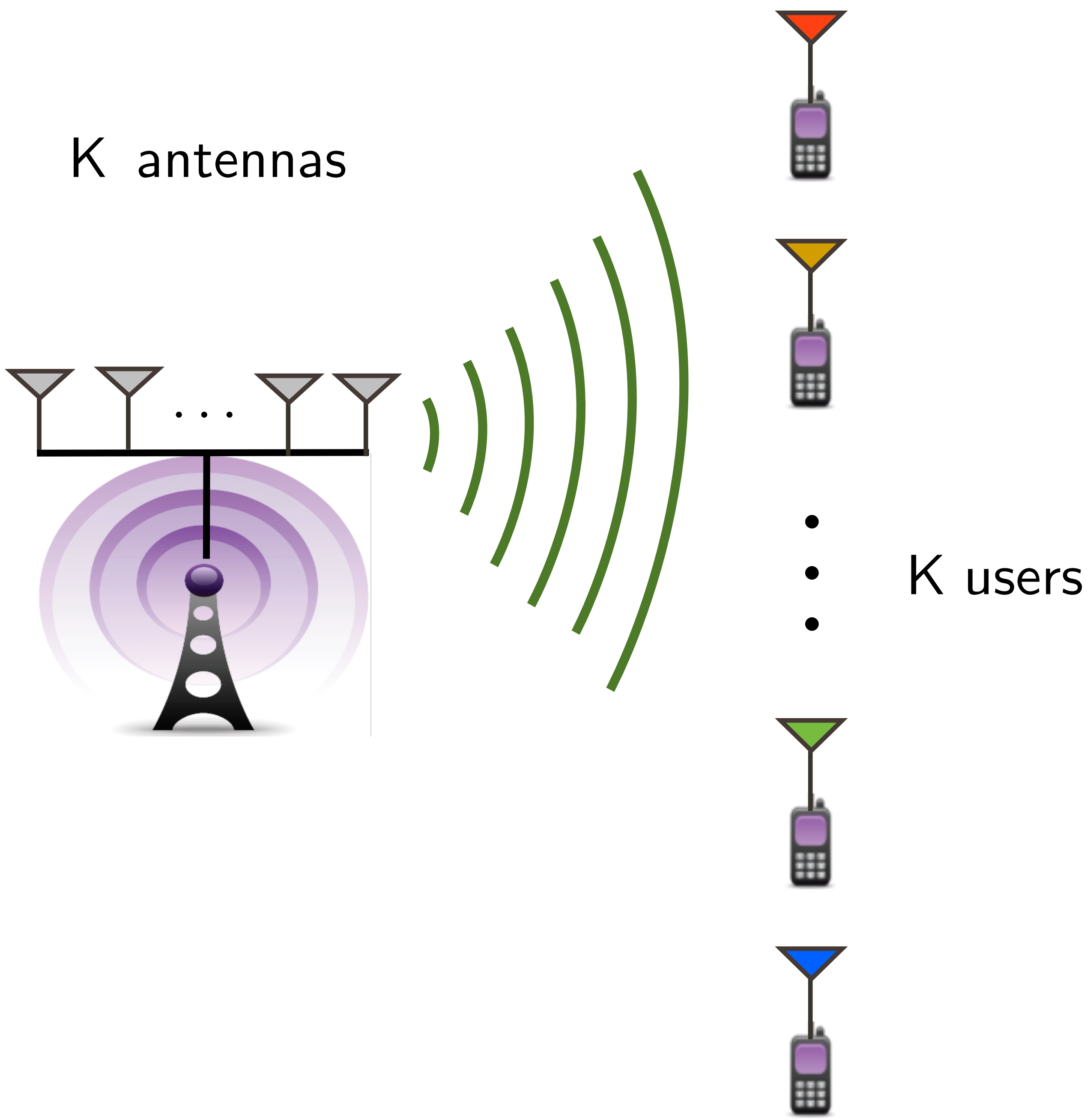
Perfect Channel Knowledge

Degrees of Freedom $= K$

No Channel Knowledge

Degrees of Freedom $= 1$

Focus: K-user Downlink MISO



Perfect Channel Knowledge

Degrees of Freedom $= K$

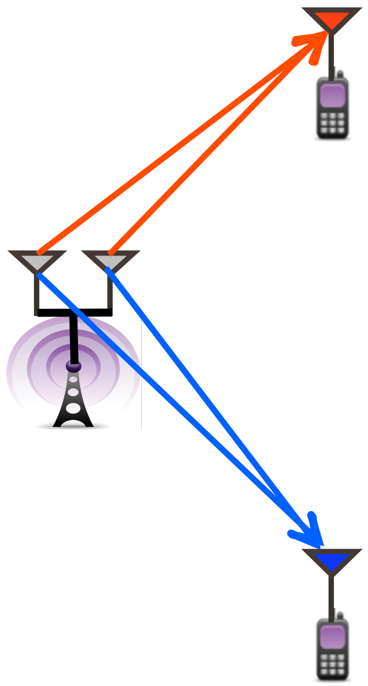
Delayed Channel Knowledge

?

No Channel Knowledge

Degrees of Freedom $= 1$

Basic Model: Two-user Downlink MISO



Perfect Channel Knowledge– DoF $= 2$

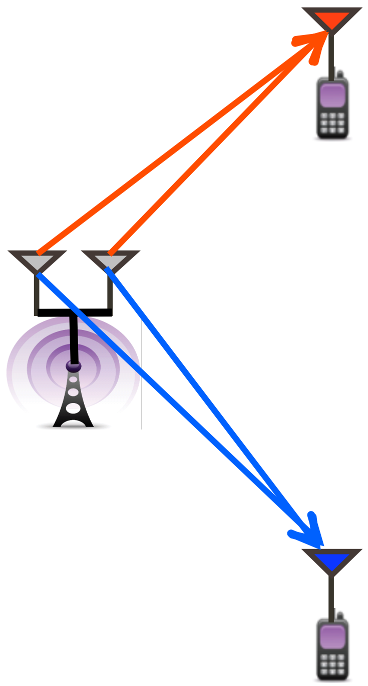
Delayed Channel Knowledge

?

No Channel Knowledge– DoF $= 1$

Basic Model: Two-user Downlink MISO

Perfect Channel Knowledge– DoF $= 2$



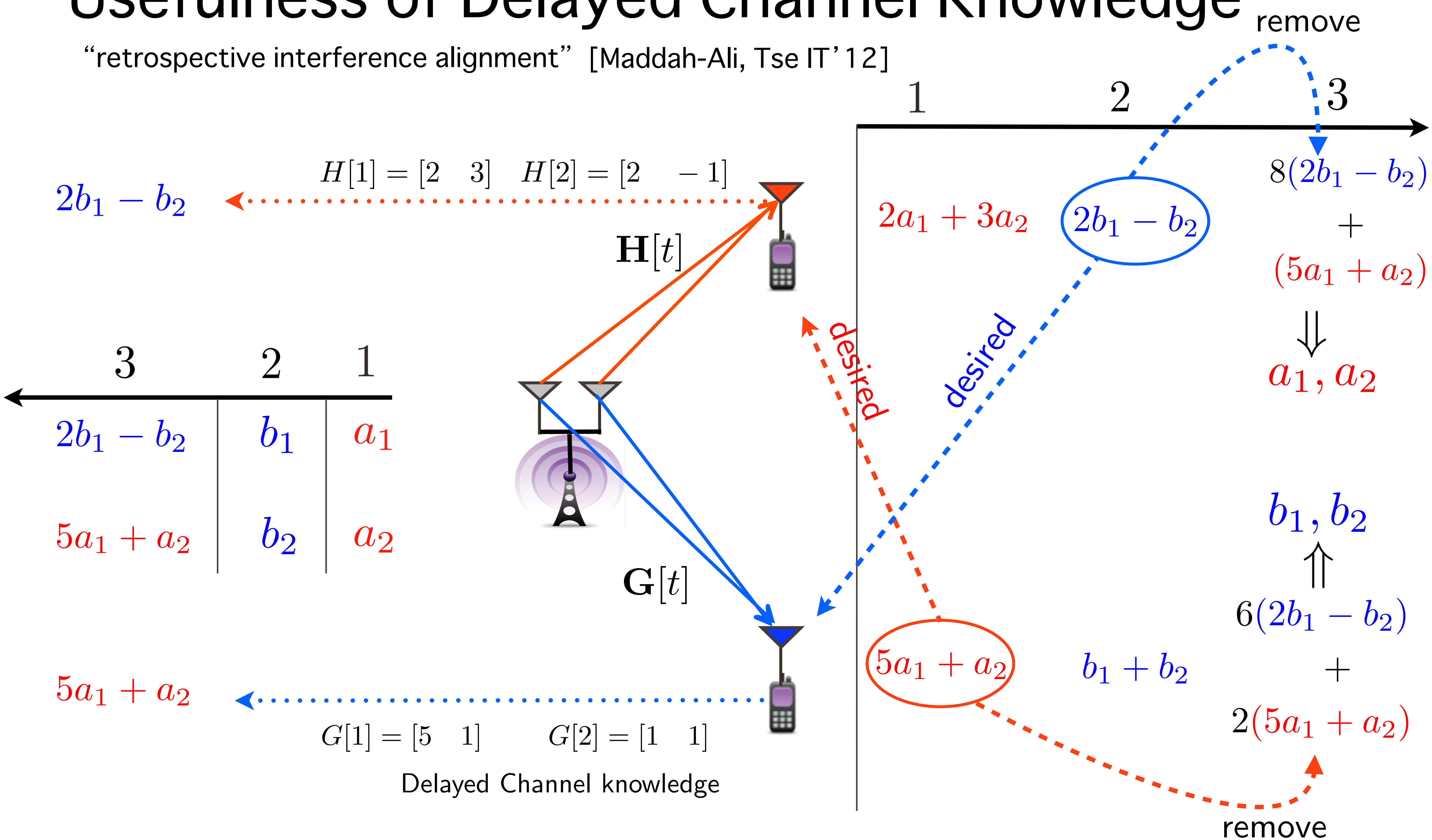
Delayed Channel Knowledge– DoF $= 4/3$

[Maddah-Ali, Tse IT'12]

No Channel Knowledge– DoF $= 1$

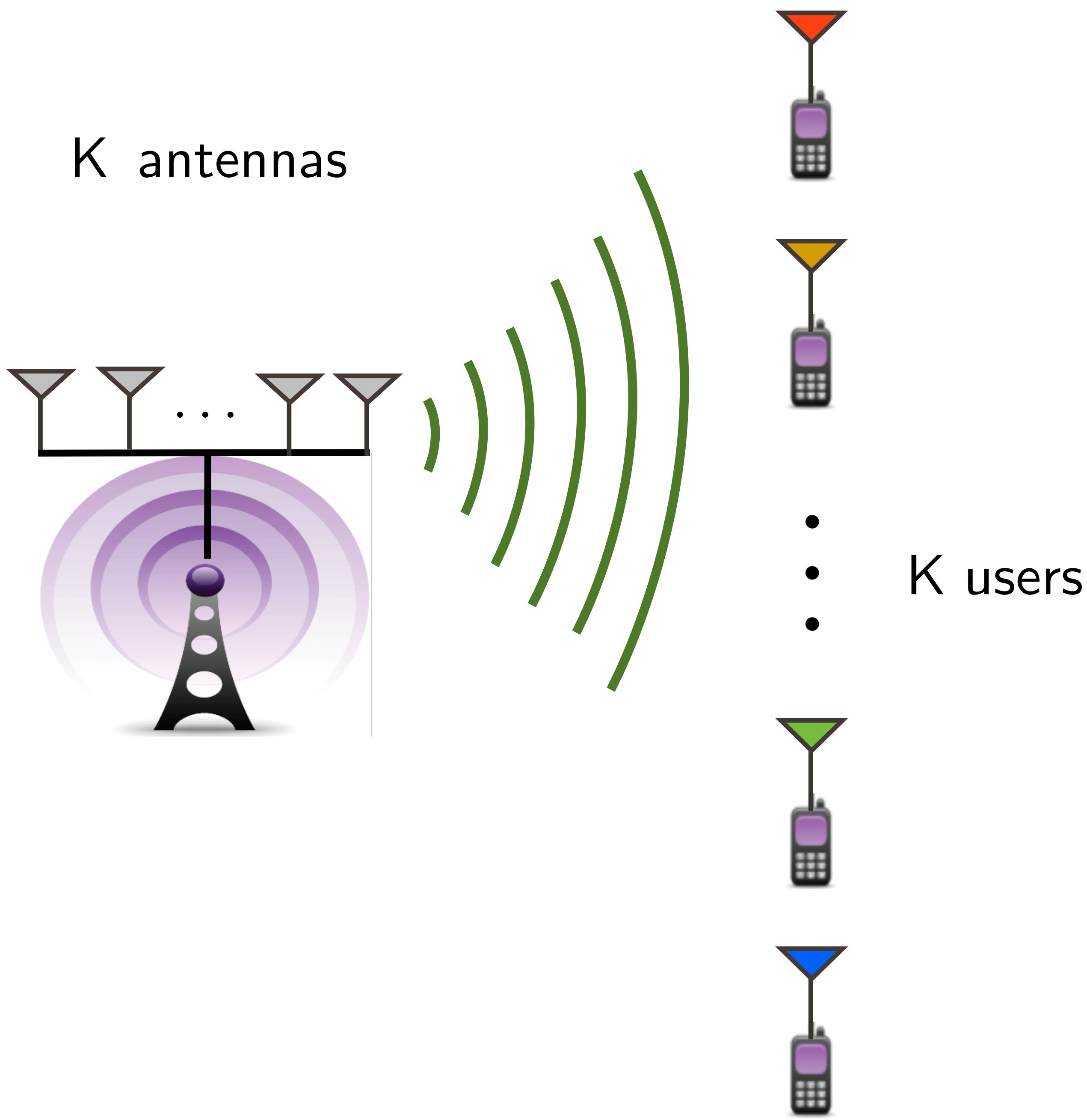
Usefulness of Delayed Channel Knowledge

"retrospective interference alignment" [Maddah-Ali, Tse IT'12]



Degrees-of-Freedom $= \frac{4}{3}$ 33% gain!

K-user Downlink MISO



Perfect Channel Knowledge

Degrees of Freedom $= K$

[Maddah-Ali, Tse IT'12]

Delayed Channel Knowledge

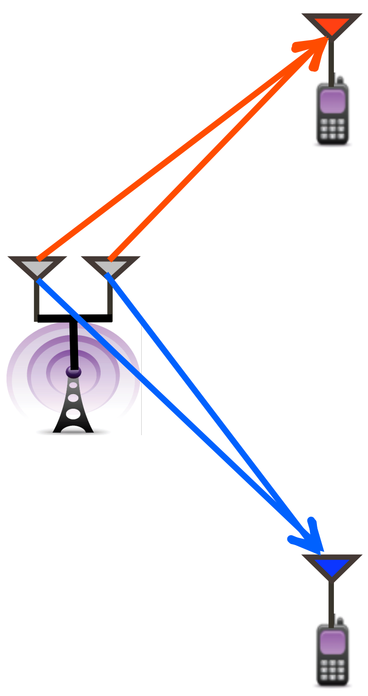
$$\text{Degrees of Freedom} = \frac{K}{1+\frac{1}{2}+\ldots+\frac{1}{K}}$$

$$\approx \frac{K}{\log(K)}$$

No Channel Knowledge

Degrees of Freedom $= 1$

Returning to the Two-user Downlink MISO



Perfect Channel Knowledge– DoF $= 2$

(from both users)

Delayed Channel Knowledge– DoF $= 4/3$

(from both users)

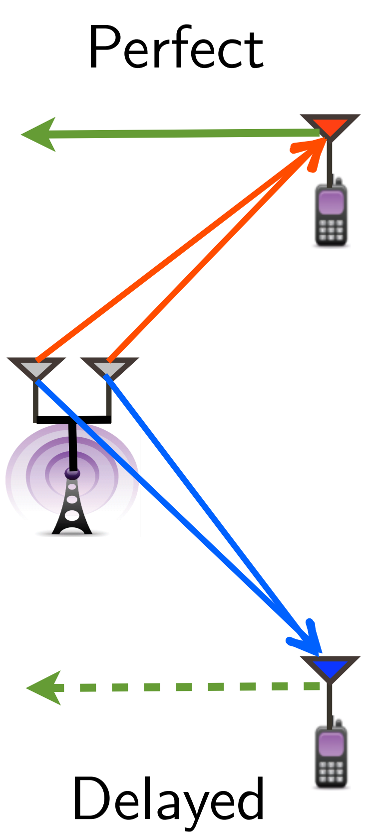
No Channel Knowledge– DoF $= 1$

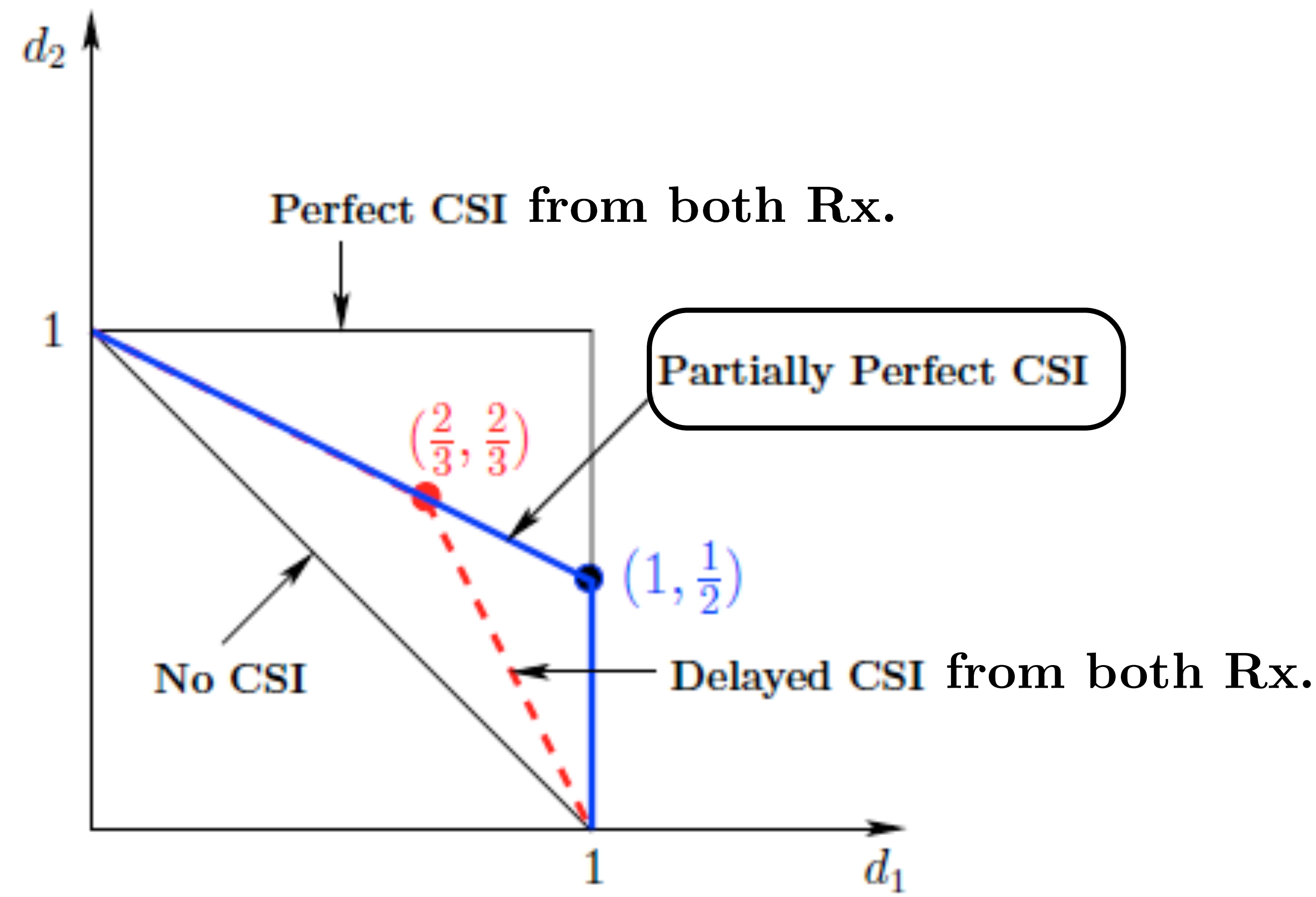
In practice, feedback quality and delay may vary across users.

Heterogenous Channel Knowledge

[Tandon, Maddah-Ali, Tulino, Poor, Shamai - ISWCS'12]

Feedback quality & delay can vary across users.

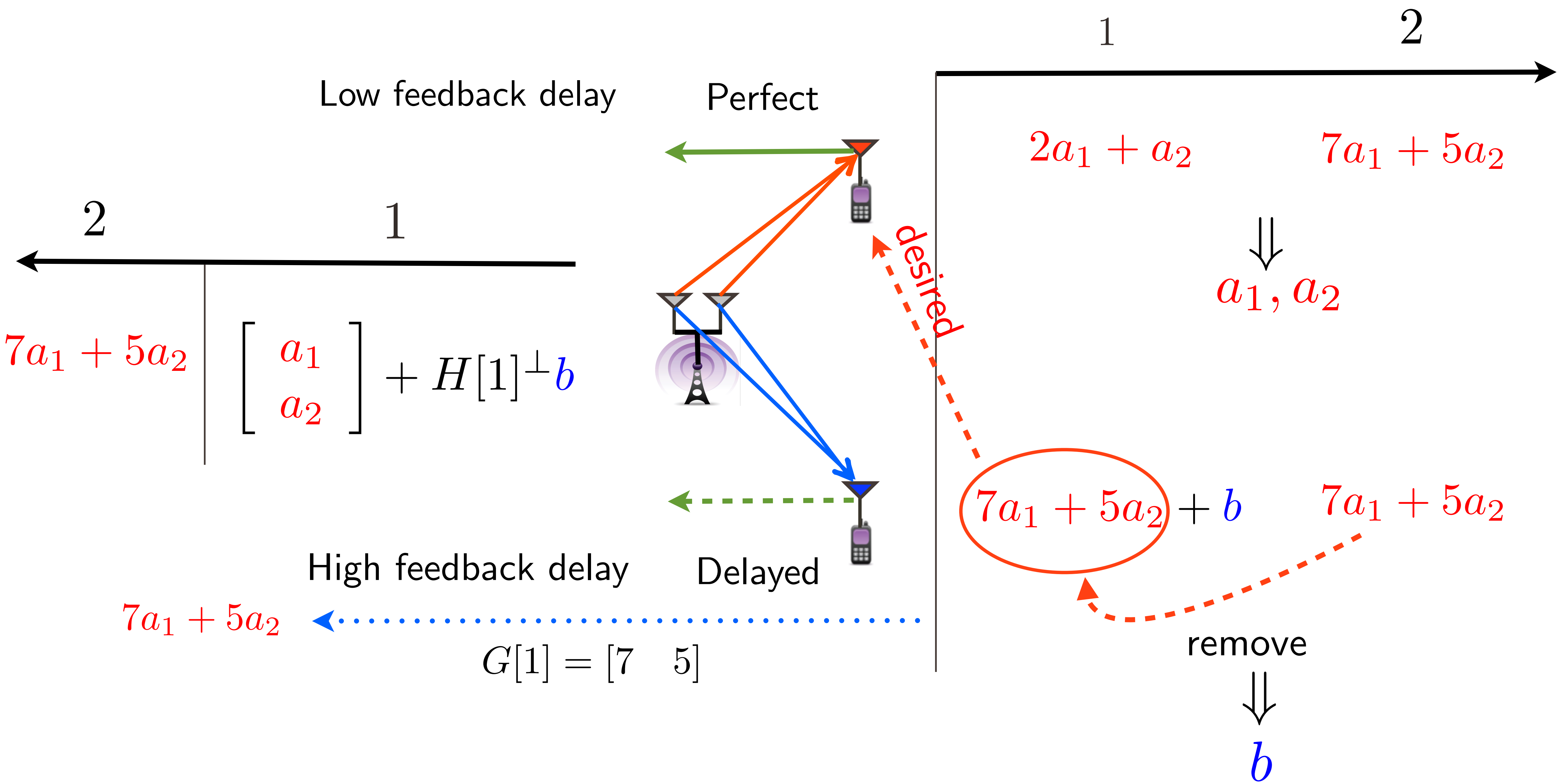




Maximum sum-DoF is at (1, 1/2) with partially perfect CSI.

Achieving Maximum Sum-DoF of 3/2

[Maleki-Jafar-Shamai, JSTSP'12]



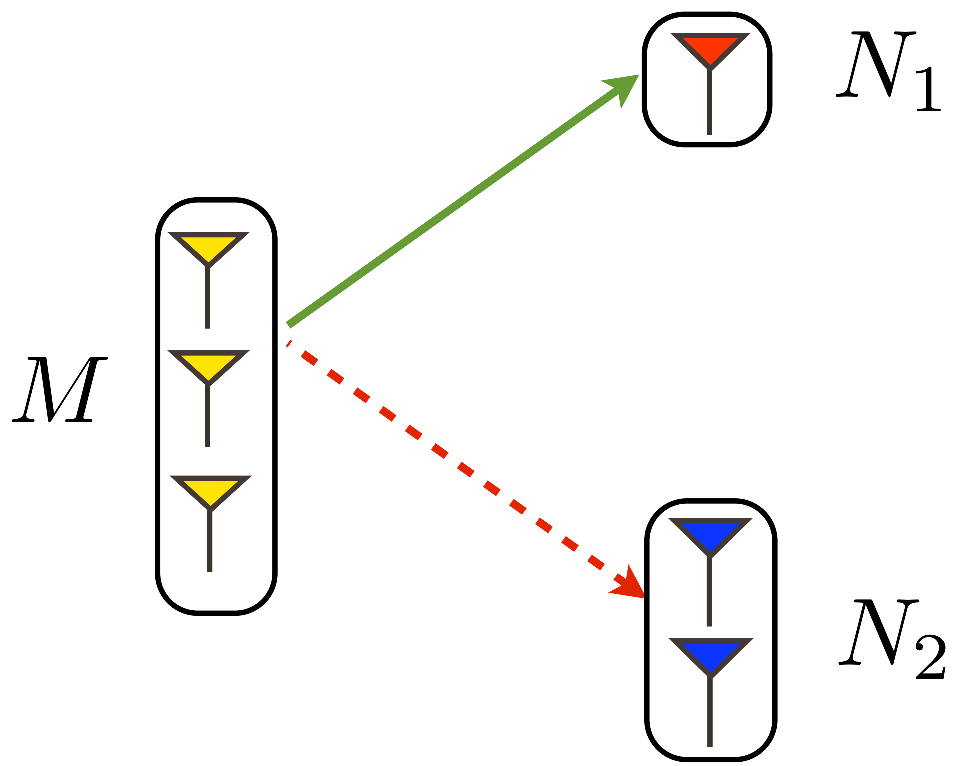
Degrees-of-Freedom $= 3/2$

Heterogeneous Channel Knowledge: General Result

[Tandon, Maddah-Ali, Tulino, Poor, Shamai - ISWCS'12]

DoF Region of (M, N_1, N_2) MIMO BC with Partial CSI

Perfect CSI from Rx 1.



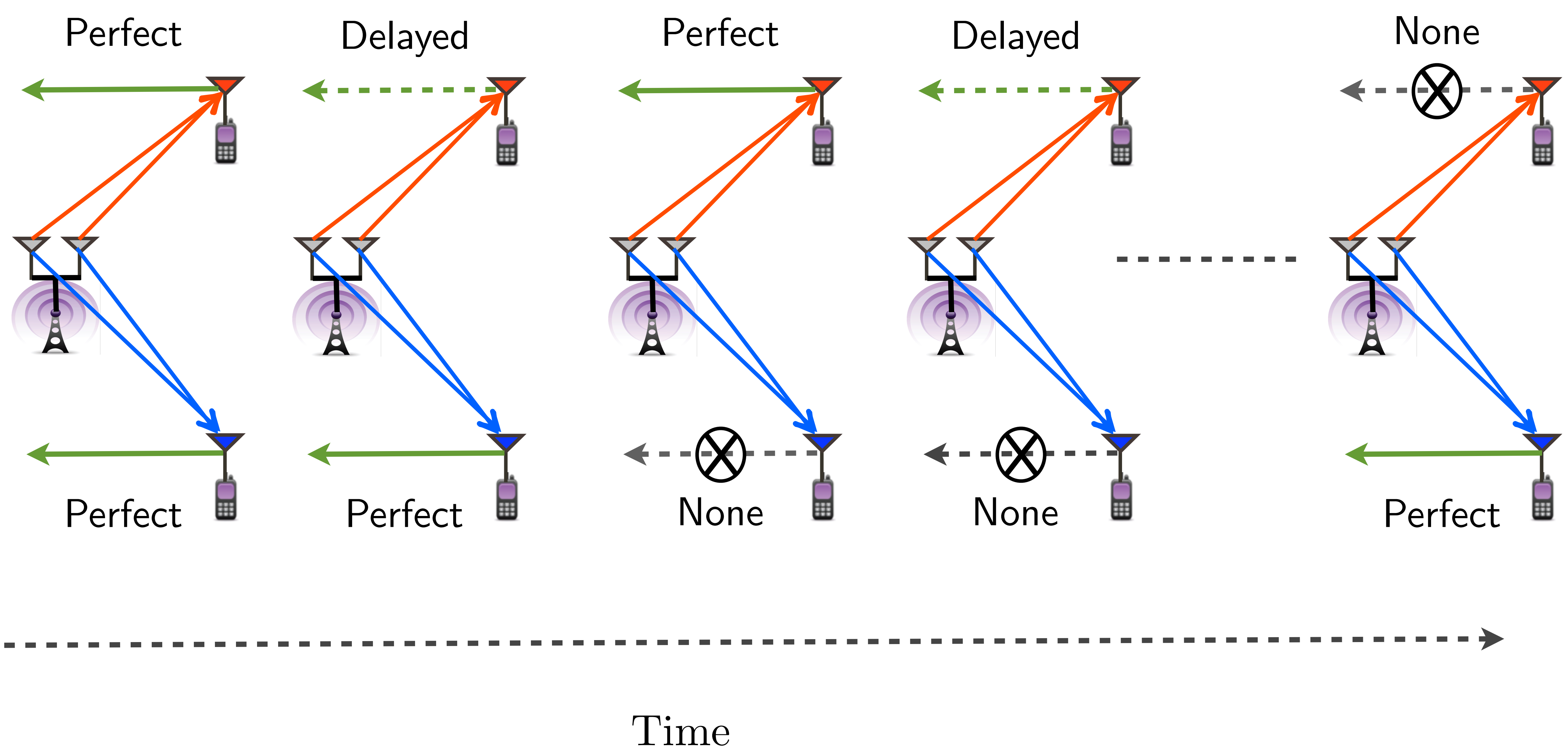
Delayed CSI from Rx 2.

$$d_1 \leq \min(M, N_1)$$

$$\frac{d_1}{\min(M, N_1 + N_2)} + \frac{d_2}{\min(M, N_2)} \leq 1.$$

Spatio-temporal Variation: Alternating CSIT

Feedback quality/delay can vary across users and over time:

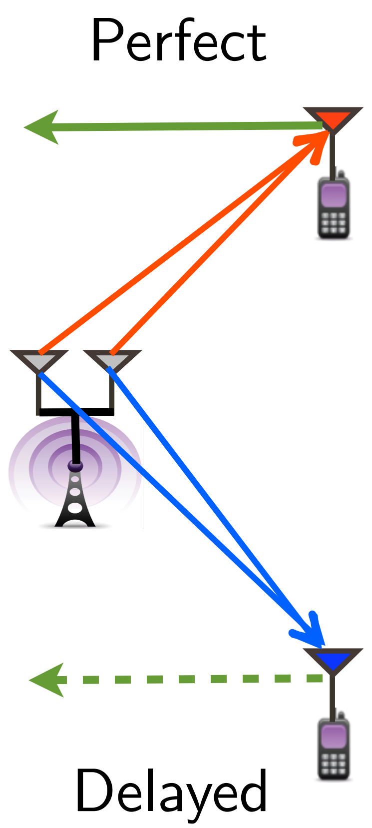


Alternating CSIT

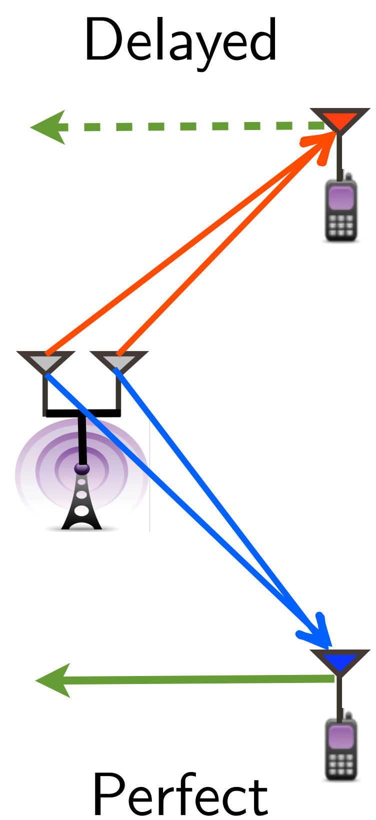
- Motivation:
  - Time-varying nature of wireless channels
  - Feedback frequency can vary across users and in time
  - CSIT acquisition can be deliberately varied (as a design parameter)
- Challenges & Benefits:
  - Some non-alternating problems are open (optimal DoF not known)
  - Can be solved under the lens of alternating CSIT
  - Alternation can provide significant gains

An Example: P-D and D-P

$\text{Optimal DoF} = \frac{3}{2}$



$\frac{2}{3}$rd fraction of time.



$\text{Optimal DoF} = \frac{3}{2}$

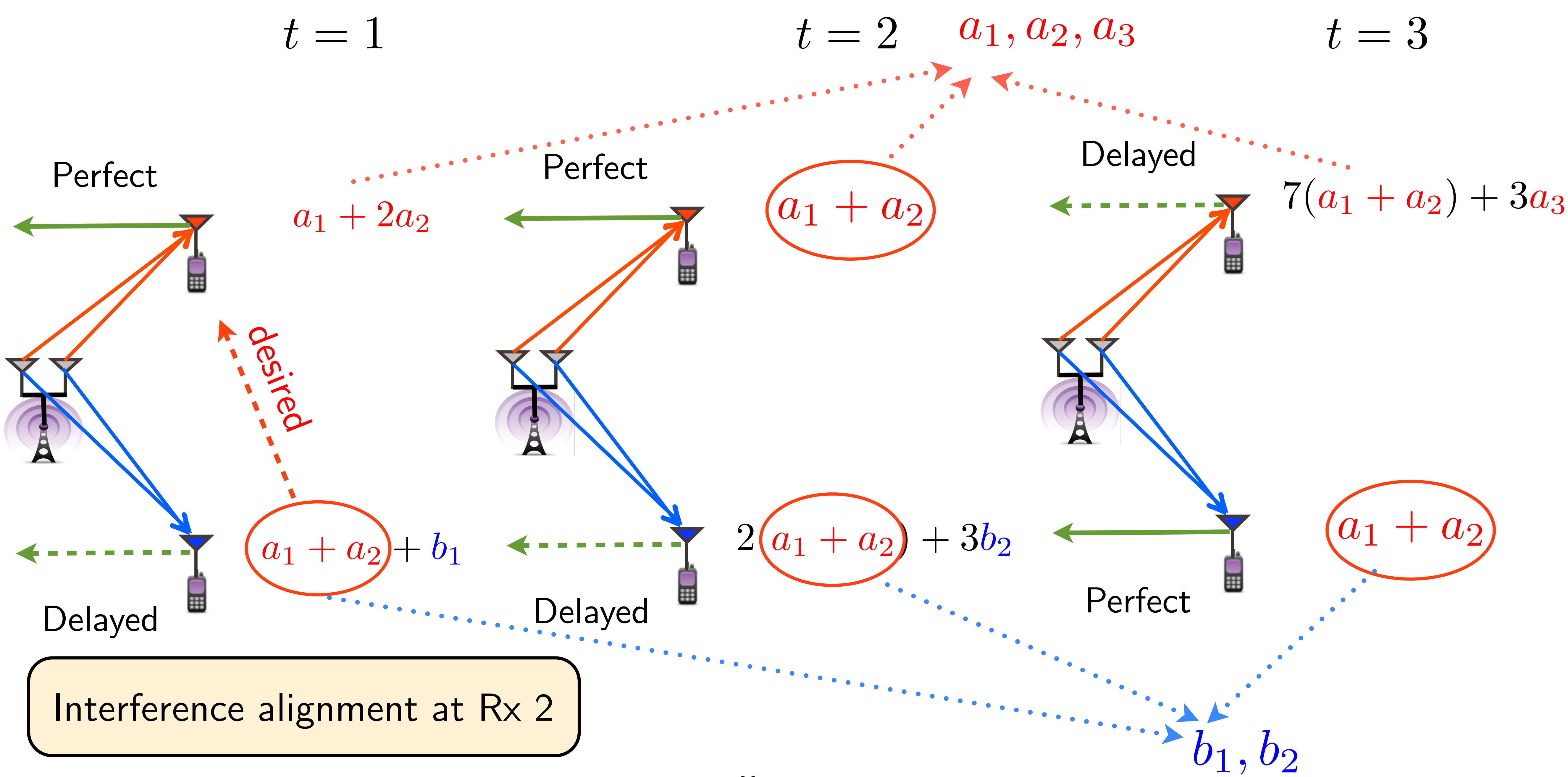
$\frac{1}{3}$rd fraction of time.

We ask: what is the optimal DoF ?

Clearly optimal DoF $\geq \frac{2}{3} \times \frac{3}{2} + \frac{1}{3} \times \frac{3}{2} = \frac{3}{2}$

$$\text{Optimal DoF} = \frac{5}{3} \qquad 44\% \text{ gain}$$

Key Idea: Code Across Multiple CSIT States



Interference alignment at Rx 2

Degrees of Freedom $= \frac{5}{3}$ 44% gain beyond $3/2$

General Result: Alternating CSIT

[Tandon-Jafar-Shamai-Poor - IT'13]

- 9 States: PP, PD, DP, PN, NP, DN, ND, DD, NN

Fraction of occurrence $\lambda_{I_1 I_2}$; $I_1, I_2 \in \{P, D, N\}$

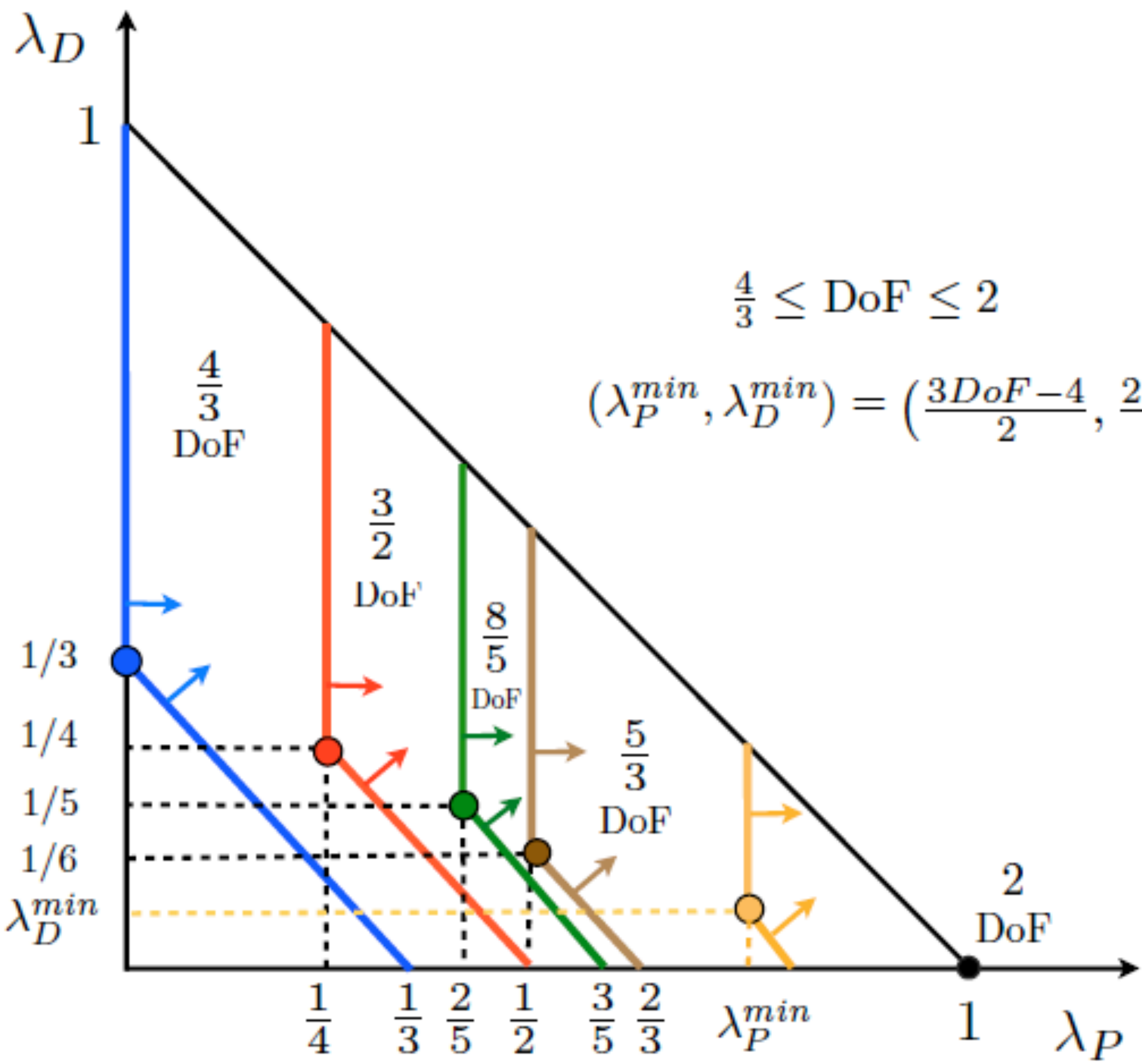
$$\sum_{I_1, I_2} \lambda_{I_1 I_2} = 1 \qquad \lambda_{I_1 I_2} = \lambda_{I_2 I_1}$$

$$\begin{aligned} d_1 &\leq 1 \\ d_2 &\leq 1 \\ d_1 + 2d_2 &\leq 2 + \lambda_P \\ 2d_1 + d_2 &\leq 2 + \lambda_P \\ d_1 + d_2 &\leq 1 + \lambda_P + \lambda_D \end{aligned}$$

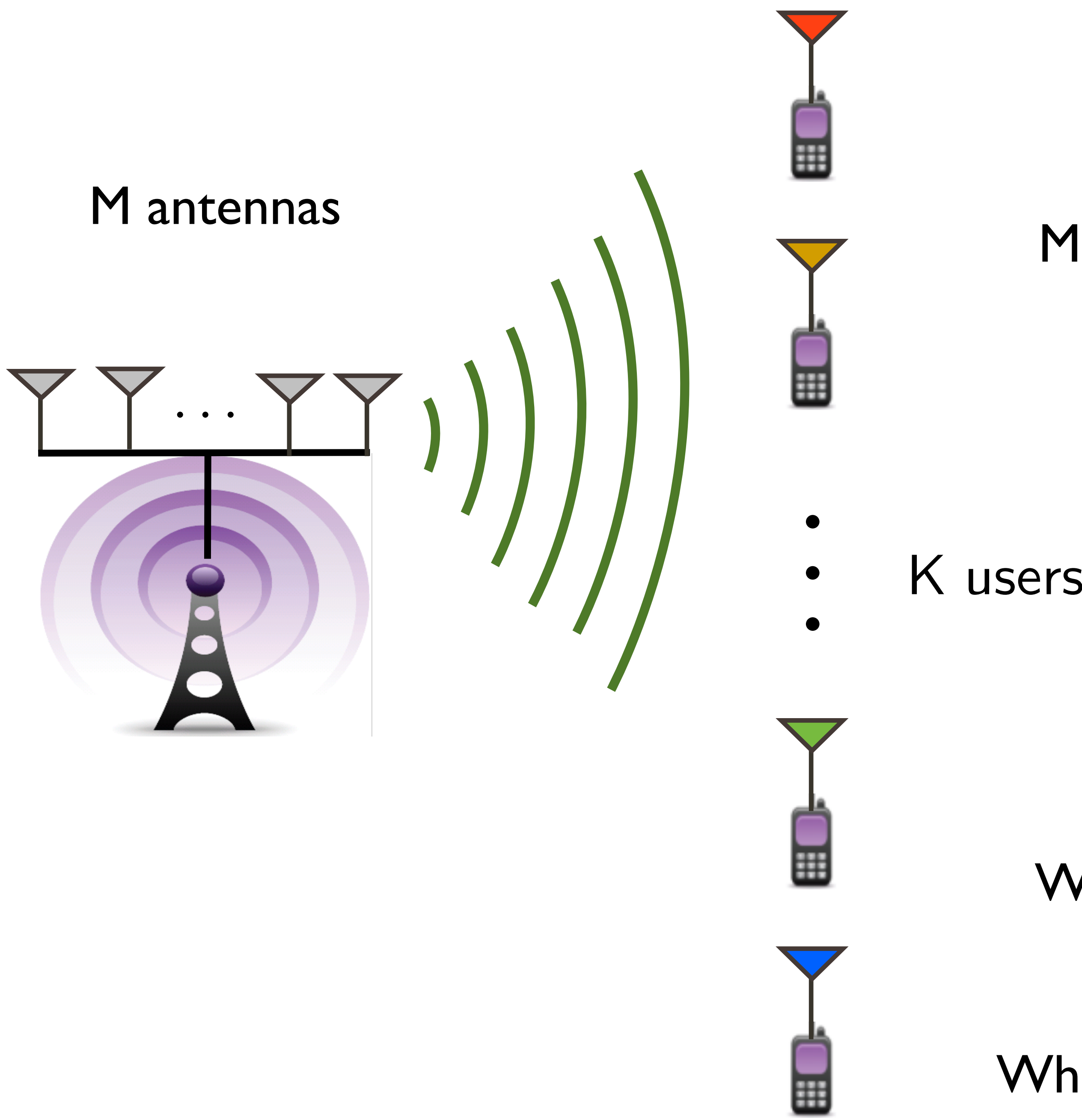
$$\lambda_P \triangleq \lambda_{PP} + \lambda_{PD} + \lambda_{PN}$$
$$\lambda_D \triangleq \lambda_{DD} + \lambda_{PD} + \lambda_{DN}.$$

Tradeoff: Delayed vs Perfect Knowledge

[Tandon-Jafar-Shamai-Poor - IT'13]



Extension: K-user Downlink MISO



Maximum possible sum DoF = min(M, K)

Minimum perfect CSIT to achieve maximum sum DoF:

$$\lambda^*(M,K) = \begin{cases} 0, & \min(M,K) = 1 \\ \frac{\min(M,K)}{K}, & \min(M,K) > 1. \end{cases}$$

Open problems:

What is the minimum perfect CSIT to achieve arbitrary DoF ?

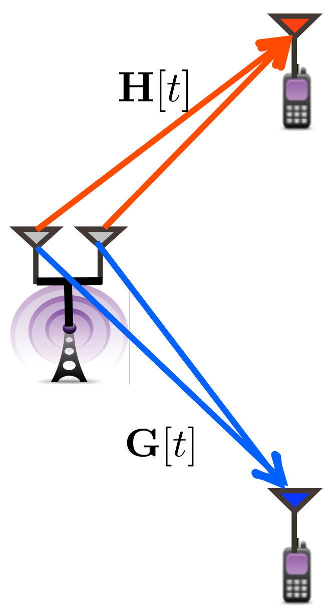
What are the tradeoffs among perfect/delayed/no CSIT ?

Beyond Delayed Channel Knowledge

Beyond Delayed Channel Knowledge

Transmit

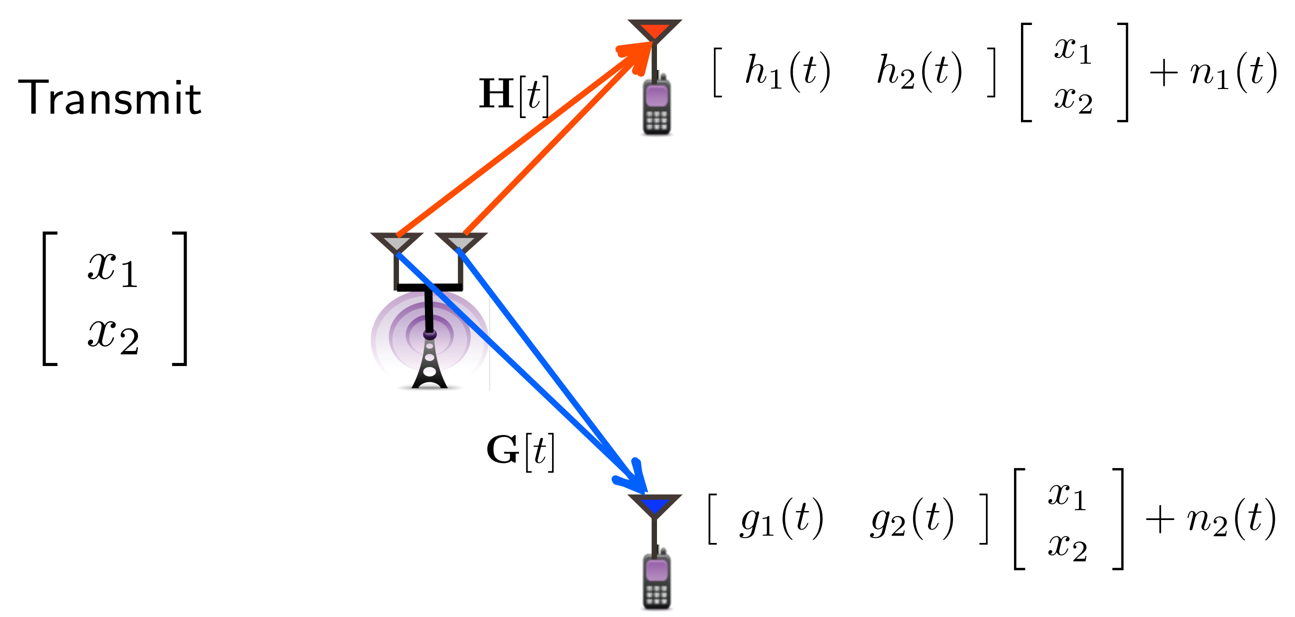
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



$$\begin{bmatrix} h_1(t) & h_2(t) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1(t)$$

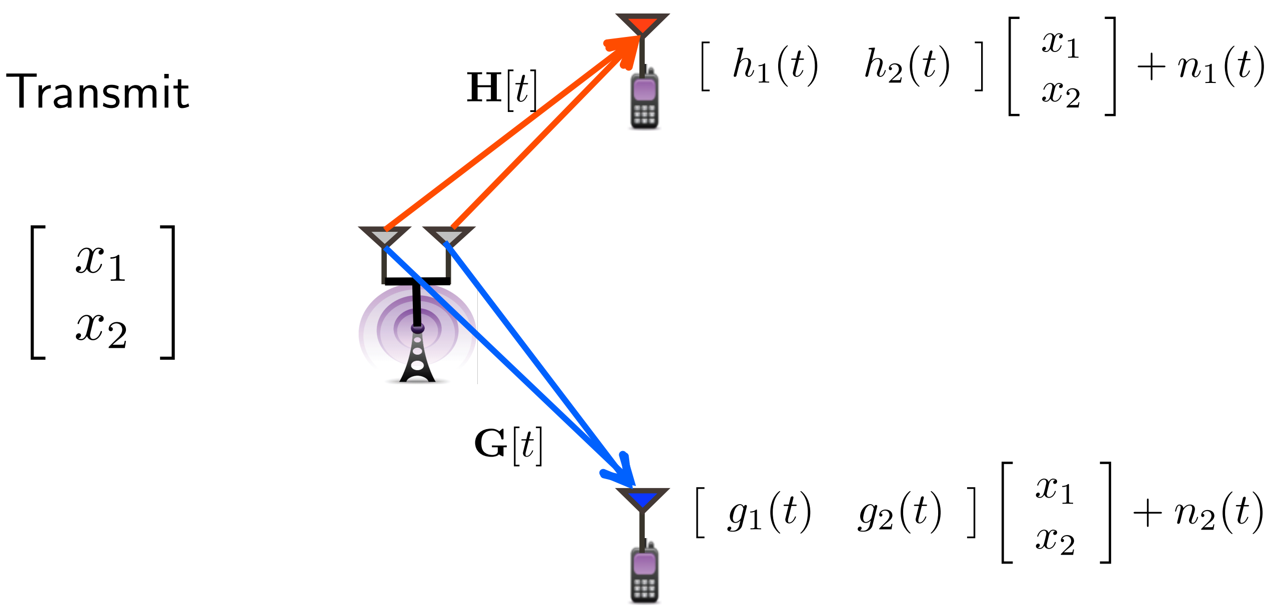
$$\begin{bmatrix} g_1(t) & g_2(t) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2(t)$$

Beyond Delayed Channel Knowledge



If, in addition to channel state, transmitter also has outputs ... does DoF increase?

Beyond Delayed Channel Knowledge



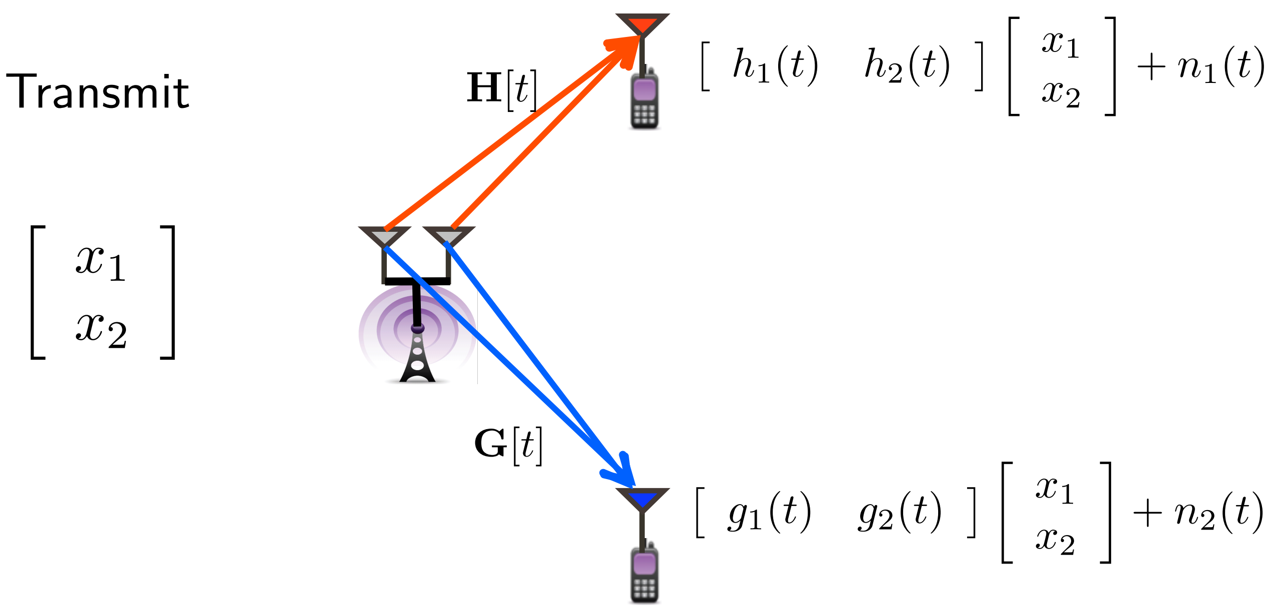
If, in addition to channel state, transmitter also has outputs ... does DoF increase?

Answer: No!

Output Feedback + Delayed CSI = Delayed CSI

[Maddah-Ali, Tse IT'12]

Beyond Delayed Channel Knowledge



If, in addition to channel state, transmitter also has outputs ... does DoF increase?

Answer: No!

Output Feedback + Delayed CSI = Delayed CSI

[Maddah-Ali, Tse IT'12]

(But for the MIMO interference channel the answer is yes.)

[Tandon-Mohajer-Poor-Shamai, IT'13]

Summary: MISO Fading Broadcast Channels

- Channel state information via feedback
- Retrospective interference alignment
- Advantages of spatio-temporal variability of channel knowledge

Summary

- Background
  - Point-to-point channels
  - Multi-terminal channels
- Static Interference Channels
  - Why feedback helps
  - Feedback gain for many-user interference channels
- Fading MISO Broadcast Channels
  - The effects of channel state feedback
  - Spatio-temporal variation in channel state feedback