IEEE Information Theory Society Newsletter

Vol. 62, No. 6, June 2012 Editor: Tara Javidi ISSN 1059-2362

President's Column

Muriel Médard

The writing of this column has been marked by many different emotions. When I began composing my message, I was thinking about continuing our reflection on our Society in the context of our IEEE review and of the upcoming ISIT. Having attended the TAB meeting in February, where I attended our Transactions' glowing review, and being in the midst of preparing for ISIT in Cambridge, I was trying to distill for this column the promises and challenges that lie before us. Before I was able to commit my thoughts to text, the untimely death of our colleague and friend Tom Cover made me realize that my message could not be about the future without being, foremost, about the past.

This column will not be an eulogy to Tom—there are many far better qualified than I am to write such an article. Our Society extends its deepest condolences to Tom's family and friends. Our thoughts particularly turn to Karen, whom many of us know well from her participation in the life of the Society. There will be events commemorating Tom, both within the Society (we are planning an event at ISIT in Cambridge) and at Stanford (please refer to the announcement in the newsletter on this matter). In reflecting on our loss of Tom, it struck me that he embodied the best of our Society, both in mind and heart.

That Tom was, to quote our Senior Past President Frank Kschischang, an "information theory giant", is unquestionable. Tom's book opened information theory to generations of students. Most of us have pored over his fundamental papers, trying to glean some of his insight and intuition, or at least capture in our own thinking traces of his intellectual elegance. Yet he remained humble and kind. He treated all as peers, earnestly discussing technical points with colleagues and students alike. I recall chatting with him in his office while, in a scheme that seemed to epitomize random access, fellow faculty and students from various groups came and went with their own questions and comments. All were greeted with his hallmark warmth and humor.

He was also a long-time member of our Society, who valued service to the Society, and recognized that the strength of the Society lies not only in the technical contributions of the individual members, but also in the community it provides to all of us. I had the pleasure and privilege to serve on our Board of Governors with Tom for several years. I marveled at how generously he shared his time and talents, and at how he took the time to encourage participation by more junior members in the governance of the Society. The fact that an academic of his stature never thought it beneath him to serve the Society in whatever capacity was needed explains in large part, I believe, the strength of our Society.

Which brings back us to the examination of the state of our Society. It is clear that the quality of our Transactions, the vibrance of our conferences, the dynamism of our Summer schools, all rely on the fact that our Society is built on a combination of intellectual excellence and of devotion to community. This dual foundation we owe to people who, like Tom, have remained engaged in the Society with both intelligence and affection. Our most fitting tribute to Tom may be to strive to preserve his legacy, by maintaining a Society built on a shared basis of scientific pursuit and friendship.

From the Editor

Tara Javidi

Dear IT Society members,

With sadness, this issue of the newsletter starts with Tom Cover's memorial information communicated by Young Han Kim and Abbas El Gamal. In addition to our regular contributions by Tony Ephremides and Solomn Golomb, we have recaps of the plenary talk by Wojciech Szpankowski at ISIT 2012.

All of us intellectually rely and depend on the IEEE Transactions on Information Theory. I would like to thank Helmut Bölcskei, the Editor-in-Chief of the Transactions, to provide a careful and detailed report on the state of the IT Transactions. This issue also includes reports on the first Software Radio Implementation Forum (SRIF) and the recent activities of the IEEE IT student committee, kindly submitted by Soung Liew and Elza Erkip. Last but not least, we have a call for participation in the IT society mentoring program from the outreach committee.

As a reminder, announcements, news and events intended for both the printed newsletter and the website, such as award announcements, calls for nominations and upcoming conferences, can be submitted jointly at the IT Society website http://www.itsoc.org/, using the quick links "Share News" and "Announce an Event." Articles and columns also can be e-mailed to me at ITsocietynewsletter@ece.ucsd.edu with a subject line that includes the words "IT newsletter." The next few deadlines are:

Issue	Deadline
September 2012	July 10, 2012
December 2012	October 10, 2012
March 2013	January 10, 2013

Please submit plain text, LaTeX or Word source files; do not worry about fonts or layout as this will be taken care of by IEEE layout specialists. Electronic photos and graphics should be in high resolution and sent as separate files. I look forward to hear your suggestions and contributions for future issues of the newsletter.

IEEE Information Theory Society Newsletter

IEEE Information Theory Society Newsletter (USPS 360-350) is published quarterly by the Information Theory Society of the Institute of Electrical and Electronics Engineers, Inc.

Headquarters: 3 Park Avenue, 17th Floor, New York, NY 10016-5997.

Cost is $1.00 per member per year (included in Society fee) for each member of the Information Theory Society. Printed in the U.S.A. Periodicals postage paid at New York, NY and at additional mailing offices.

Postmaster: Send address changes to IEEE Information Theory Society Newsletter, IEEE, 445 Hoes Lane, Piscataway, NJ 08854.

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Thomas M. Cover in Memoriam 1938–2012

Tom Cover, one of the greatest information theorists and a wonderfully inspiring teacher and mentor, passed away in Palo Alto on March 26, 2012. During his 48-year career as a professor of Electrical Engineering and Statistics at Stanford University, he graduated 63 PhD students, published over 120 journal papers in learning, information theory, statistical complexity, and portfolio theory, and coauthored Elements of Information Theory, the most widely-cited book in the field. In recognition of his seminal contributions, Tom Cover received many awards and honors, including the IEEE Richard W. Hamming Medal, the IEEE Information Theory Society Claude E. Shannon Award, and the IEEE Information Theory Paper Award. He was a Fellow of the IEEE and a member of the National Academy of Engineering and the American Academy of Arts and Sciences.

There will be a special session at the International Symposium on Information Theory (ISIT) on Monday, July 2, 2012, 8:00 PM–9:30 PM. If you would like to say a few words in his memory, please contact Denise Murphy <denise@ee.stanford.edu>.

A memorial service will be held in the afternoon of Friday, October 12, 2012 at the Arrillaga Alumni Center on the Stanford campus. For more details on this event, please visit the memoriam website http://tinyurl.com/TomCoverMemorial

The Historian's Column

Anthony Ephremides

In less than one year our Society lost two giants of our field. Jack Wolf and Tom Cover passed away leaving behind them a huge void, an immense legacy, and many mourning colleagues and friends in addition to their families. Jack and Tom have made history through their lives and so it is only fitting to reminisce about them in this column.

I will not review here their technical work. Their accomplishments are well documented in their published works and the minds of those who have studied them. Their students and collaborators populate our Society at all levels. Both were Shannon Award winners and both dominated the field for over forty years. Although there are many who carry on in their tradition, they are simply irreplaceable.

I will rather recall here, as a tribute to their memory, some special moments and vignettes from their lives, as I experienced them, which show their human traits that made them so dear to us all. Both Jack and Tom had above all a unique sense of humor. Each in his own way, and underneath even their most solemn moments, displayed a twinkle of levity and warmth that none could emulate.

Let me start with Jack. As everyone who has met him remembers, Jack was capable of presenting a somber and serious expression on his face that could create sometimes concern and, even, foreboding. You might feel a threat of criticism or disapproval when he put his grave face on. Yet, within an instant, a slight set of lines around the eyes and the mouth would appear, and a set of white teeth would sparkle, as all the seriousness dissolved into a broad smile and a face beaming with delight, kindness, and a hilarious mood. His laughter was contagious and his jokes innumerable and delightful.

I have recorded before his legendary performance with Jim Massey at one of the NATO Advanced Study Institutes in England when he teased Jim and staged a medieval "fight" with him in front of the incredulous eyes of the audience. And I may have mentioned other light moments involving his sharp humor and inventiveness. Here I'd like to recall a couple of them.

In 1990, he and I, along with a broad delegation of the IEEE found ourselves touring South America. The trip was complex and interesting, revealing untold aspects of life in that part of the work. At one point we were all put in a tour bus that proved to be a disaster. A rather boisterous and incompetent guide was taking us from

vendor to vendor to hopefully provide business for his friends (instead of taking us to the advertised sites). We were all irritated but helpless and endured the long hours of driving around as well as we could. The guide was constantly talking on the mike, saying inane things, thus adding to the frustration. At one point he tried to show that he was smart and taking good care of us by pointing out that he was taking us to only the best sites. He went on to claim that he avoided those who might take advantage of us. He said, “what do you think? We are not stupid!”. At which point Jack offered: “no, no! We are!”. The ensuing reaction by everyone (i.e. hearty laughter) helped restore a decent mood in our group.

At another NATO Advanced Study Institute in Southwestern France in 1983, the attendees were invited by the landlady, who had made her castle available for the meeting, to a pre-dinner reception of fine hors d’oeurvres and aperitives in the grand reception hall of her residence. The atmosphere was almost 17th century royal French, and everyone was somewhat overwhelmed. I recall taking one of the “fruit-glaces” into my mouth and just as I closed my jaw, the bite cut the fruit the wrong way forcing the pit of the fruit right out of my mouth and onto the glistening wooden floor. I was highly embarrassed and humiliated especially since, as one would expect in such a situation, everyone pretended they didn’t notice. And then my eyes crossed Jack’s. He had an almost comical expression on his face, as he looked at me with sympathy, understanding, and encouragement. Without saying a word, he managed to restore my composure and overcome the shock of the incident.

Tom’s humor was different. There was occasionally a touch of well-intentioned sarcasm in his remarks, a lot of self-depreciation, a unique propensity for two-sided, double-meaning comments. I know I have reported in the past the occasion where, during a plenary talk at the ISIT in St. Lovite, Tom went through some intermittent dozing-offs as he had been caught up with fatigue and jet lag. A young student who observed him, but had no idea who he was, thought he’d try a pleasantry with him and asked, after the talk was over, whether Tom had a nice nap. Tom looked at him and said dryly” “yes, except that I was awaken by some very interesting theorems”. The student looked puzzled by the response and decided, wisely, not to continue.

Tom was also a gourmet and bon-vivant. We enjoyed many fabulous dinners accompanied with fine wines in various places around the world. Yet, he was uniquely approachable and friendly, especially to students and young researchers, with who he liked to mingle encouragingly during the social functions of our symposia and workshops.

One of Tom’s highest moments of hilarity occurred several decades ago when an unnamed program director of NSF was requiring lengthy, unproductive meetings of the PI’s he was supporting. Many had tried to find a polite way to express their displeasure. And then Tom decided to intervene. He spoke with a metaphor. He told of the farmer who was cleaning his pig every time it soiled itself from rolling over in the mud. When asked why he spent so much time doing that instead of waiting to do it only at the end of the day before placing the pig in the pen, the farmer said: “what is time to a pig?”. Needless to say that the Program Director did not speak to Tom again!

I have fond memories of sharing a dinner with Jack in December of 2010 at the Globecom in Miami and with Tom at the ITA in San Diego this past February. They both were their usual selves, looking healthy, and up-beat. I felt privileged. And I, along with everyone else, will miss them dearly.

Algorithms, Combinatorics, Information, and Beyond

Plenary Talk at ISIT 2011, St. Petersburg, Russia, April 7, 2012

*Wojciech Szpankowski**
Department of Computer Science, Purdue University
West Lafayette, IN 47907 U.S.A.
spa@cs.purdue.edu

Abstract–Shannon information theory aims at finding fundamental limits for storage and communication, including rates of convergence to these limits. Indeed, many interesting information theoretic phenomena seem to appear in the second order asymptotics. So we first discuss precise analysis of the minimax redundancy that can be viewed as a measure of learnable or useful information. Then we highlight Markov types unveiling some interesting connections to combinatorics of graphical enumeration and linear Diophantine equations. Next we turn our attention to structural compression of graphical objects, proposing a compression algorithm achieving the lower bound represented by the structural entropy. These results are obtained using tools of analytic combinatorics and analysis of algorithms, known also as *analytic information theory*. Finally, we argue that perhaps information theory needs to be broadened if it is to meet today's challenges beyond its original goals (of traditional communication) in biology, economics, modern communication, and knowledge extraction. One of the essential components of this perspective is to continue building foundations in better understanding of temporal, spatial, structural and semantic information in dynamic networks with limited resources. Recently, the National Science Foundation has established the first *Science and Technology Center on Science of Information* (CSoI) to address these challenges and develop tools to move beyond our current understanding of information flow in communication and storage systems.

1. Introduction

It is widely accepted that the information revolution started in 1948 with the publication of Shannon "A Mathematical Theory of Communication". It not only inaugurated a new research field, that of information theory, but also paved the way to today's technological advances in storage and communication such as CDs, iPod, DVD and the internet. Shannon accomplished it all by first introducing a mathematical definition of information that quantifies the extent to which a recipient of data can reduce its statistical uncertainty, and then formulating two fundamental results giving us a lower bound for compression and an upper bound for reliable communication. Furthermore, Shannon declared "these semantic aspects of communication are irrelevant", somewhat abandoning his own dictum in the rate distortion theory (e.g., the distortion measure of audio is incompatible with image compression).

In this article we shall follow another Shannon commandment [66] "it is hardly to be expected that one single concept of information would satisfactorily account for (all) possible applications". So we shall argue that information theory may benefit by expanding its original goals to meet today's challenges in biology, economics, modern communication, and knowledge extraction from massive datasets (see also [1]). For this to happen more foundational work in better understanding of temporal, spatial, structural and semantic information is essential.

We are all aware of Shannon warning in his "bandwagon" paper [67] where he thundered "Information theory has, in the last few years, become something of a scientific bandwagon." It is no wonder that some developments in the 50's irked Shannon. Let us just look at the early application of information theory, say to biology. Henry Quastler launched information theory in biology in 1949 (just a year after Shannon's landmark paper and four years before the inception of molecular biology shaped by the work of Crick and Watson) in the paper written together with Dancoff "The Information Content and Error Rate of Living Things". Continuing this effort, Quastler organized two symposiums on "Information Theory in Biology". These attempts were rather unsuccessful as argued by Henry Linschitz [41], who pointed out that there are difficulties in defining information "of a system composed of functionally interdependent units and channel information (entropy) to "produce a functioning cell". To be fair, we need to point out that in 70's Manfred Eigen, Nobel laureate in biochemistry opined, "the differentiable characteristic of the living systems is information. Information assures the controlled reproduction of all constituents, thereby ensuring conservation of viability. Information theory, pioneered by Claude Shannon, cannot answer this question… in principle, the answer was formulated 130 years ago by Charles Darwin." Eigen's challenge was picked up recently in two new special issues [20, 53] on information theory in molecular biology and neuroscience. The editorial of [20] concludes: "Information Theory is firmly integrated in the fabric of neuroscience research, and a progressively wider range of biological research in general, and will continue to play an important role in these disciplines."

We are now fifty years after the bandwagon paper. In today's world the dynamic flow of information is around us from biology to modern communication to economy. Many scholars argue to broaden information theory beyond its original goals of point-to-point communication and compression of sequences: Sudan and his collaborators [25, 40] suggest that the meaning of information does start to become relevant whenever there is diversity in the communicating parties and when parties themselves evolve over time. For example, when a computer attempts to communicate with a printer both parties must talk the same language in the same format (i.e., "printer driver"). This leads Sudan and his collaborators to consider communication in the setting where encoder and decoder do not agree a priori on the communication protocols, thus encoder and decoder do not understand each other. Bennett in [5] observes that from the earliest days of information theory it has been appreciated that information is not a good *message value*. He continues to propose that the value of information lies in "parts predictable only with difficulties, things

*The work of this author was supported by NSF Science & Technology Center Grant CCF-0939370, NSF Grant CCF-0830140, AFOSR Grant FA8655-11-1-3076, and NSA Grant H98230-11-1-0141.

that the receiver could figure out without being told". This led him to define the *logical depth*. However, we still do not have a good understanding of the value of information; particularly, in biology and economics. As a matter of fact, in biology, P. Nurse in his 2008 paper [55] claims that biology is on the crossroad and further advances may be required to understand information flow. In Nurse's own words "focusing on information flow will help to understand better how cells and organisms work … and temporal order in cell memory and reproduction are not fully understood." Furthermore, in computer science F. Brooks claims [8]: "we have no theory that gives us a metric for the information embodied in structure … this is the most fundamental gap in the theoretical underpinning of information and computer science." Finally, Zeilinger goes even further in [9, 85] claiming that reality and information are two sides of the same coin, that is, they are in a deep sense indistinguishable. In communication it is widely accepted that understanding (value and flow of) temporal information is the key to further advances in computer communication [29] and wireless ad-hoc networks [26].

As the matter of fact, in recent decades the information theory community has been pursuing post-Shannon challenges as witnessed in [17, 20, 26, 29, 33, 45, 53, 58, 54, 79], to mention a few. To continue on this path, we propose two approaches that include short(er)-term and long-term research goals:

i) **Back off from infinity:** Following Ziv's 1997 Shannon Lecture, we propose to extend Shannon findings to finite size data structures (i.e., graphs, sets, social networks), that is, develop *information theory of data structures* beyond first-order asymptotics. We shall argue (see Section 2) that many interesting information-theoretic phenomena appear in the second-order terms. *Analytic information theory*—which applies complex-analytic tools to information theory—is particularly suited for such investigations. We illustrate it in the next section by studying the minimax redundancy problem.

ii) **Science of Information:** In general, we endeavor to do some foundational work in structural, temporal, spatial and semantic information in dynamic networks with cooperating users (see also recent panel discussion [1]). We also argue that we need a better understanding of complex systems with representation-invariant information. In Section 3 we describe some attempts towards this goal.

In 2010 the National Science Foundation established the first Science and Technology Center for Science of Information (http://soihub.org) "*to advance science and technology through a new quantitative understanding of the representation, communication and processing of information in biological, physical, social and engineering systems*." The center is located at Purdue University and partner institutions include: Bryn Mawr, Berkeley, Howard, MIT, Princeton, Stanford, Texas A&M, UIUC, and UCSD. Some specific Center goals are to: (i) define core theoretical principles governing transfer of information; (ii) develop meters and methods for information; (iii) apply science of information to problems in physical and social sciences, and engineering; and (iv) offer a venue for multidisciplinary long-term collaborations.

The plan for the paper is as follows. In the next section we discuss the maximal minimax redundancy for memoryless, Markovian, and renewal sources solved by analytic and combinatorial methods. In Section 3 we present a few problems illustrating broader science of information. In particular, we offer some new results on graphical compression as an illustration of structural information.

2. Analytic Information Theory

Jacob Ziv in his 1997 Shannon Lecture presented compelling arguments for "backing off" from first-order asymptotics in order to predict the behavior of real systems with finite length description. To overcome these difficulties, the so called *non-asymptotic analysis*, in which lower and upper bounds are established with controllable error terms, becomes quite popular. However, we argue that developing *full asymptotic expansions* and more precise analysis may be even more desirable. Furthermore, following Hadamard's precept[1], we propose to study information theory problems using techniques of complex analysis[2] such as generating functions, combinatorial calculus, Rice's formula, Mellin transform, Fourier series, sequences distributed modulo 1, saddle point methods, analytic poissonization and depoissonization, and singularity analysis [76]. This program, which applies complex-analytic tools to information theory, constitutes analytic information theory.

Analytic information theory can claim some successes in the last decade. We mention a few: proving in the negative the Wyner-Ziv conjecture regarding the longest match [72, 73]; establishing Ziv's conjecture regarding the distribution of the number of phrases in the LZ'78 compression scheme [35, 39]; showing the right order of the LZ'78 redundancy [62, 49]; disproving the Steinberg-Gutman conjecture regarding lossy pattern matching compression schemes [50, 84, 46]; establishing precise redundancy of Huffman's code [75] and redundancy of a fixed-to-variable no prefix free code [77]; deriving precise asymptotics of minimax redundancy for memoryless sources [81, 74, 78], Markov sources [59, 37] and renewal sources [23, 21]; precise analysis of variable-to-fixed codes such as Tunstall and Khodak codes [22]; designing and analyzing error resilient Lemple-Ziv'77 data compression scheme [48], and finally establishing entropy of hidden Markov processes [64] and the noisy constrained capacity [30, 38].

In this section, we illustrate the power of analytic information theory on a few examples taken from the analysis of the minimax redundancy and enumeration of Markov types. First, however, we interpret minimax redundancy as a measure of learnable or useful information capturing regularity properties of an object.

2.1. Learnable/Useful Information and Redundancy

One of the fundamental questions of information theory and statistical inference probes how much "useful or learnable information" one can actually extract from a given data set. To shed some light on this problem, let a binary sequence $x^n = x_1 \dots x_n$ be given.

We would like to understand how much useful information, structure, regularity, or summarizing properties are in x^n. For example, for a binary sequence the number of ones is a regularity property, the positions of ones are not. Let in general S be such a summarizing property. We can describe it in two parts. First, we describe the

[1] The shortest path between two truths on the real line passes through the complex plane

[2] Andrew Odlyzko wrote: "Analytic methods are extremely powerful and when they apply, they often yield estimates of unparalleled precision."

set S, and then the location of x^n in S that requires $\log|S|$ bits (the latter is a good measure of the string complexity). We denote by $I(S)$ the number of bits describing it. Usually, S can be described in many ways, however, one should choose S so that it extracts all relevant information and nothing else. It means we need S that minimizes $I(S)$. We denote such a set as $\hat{S}$ and call it I-sufficient statistic. It makes sense to call $I(\hat{S})$ the *learnable information*.

We now consider two concrete measures of learnable information. If $\hat{S}$ is the shortest program on a universal Turing machine, then $I(\hat{S})$ becomes *Kolmogorov-information* [13] $K(\hat{S})$, and $K(x^n) = K(\hat{S}) + \log|\hat{S}|$. For example, if x^n is a binary sequence, we first describe the *type* of x^n (e.g., the number of ones) that requires $O(\log n)$ bits, and then location of x^n within the type which requires $\log\binom{n}{k} \approx nH(k/n)$ bits. While this sounds reasonable, in general Kolmogorov information is not computable, so we need another approach.

We now turn our attention to *computable* useful information contained in a sequence x^n generated by a source belonging to a class of parameterized distributions $\mathcal{M}(\Theta) = \{P_\theta : \theta \in \Theta\}$ for some k-dimensional space Θ. We follow here Rissanen [60] and Grunwald [28]. Let $\hat{\theta}(x^n)$ be the maximum likelihood (ML) estimator, that is, $\hat{\theta}(x^n) = \arg\max_{\theta\in\Theta} P_\theta(x^n)$. Observe that for a given sequence x^n, produced either by θ or by θ', we can use $\hat{\theta}(x^n)$ to decide which model generates the data with a small error probability, *provided* these two parameters are far apart in some distance. If these two models, θ and θ' are too close to each others, they are virtually indistinguishable, and they do not introduce any additional useful information. In view of this, it is reasonable to postulate that learnable information about x^n is summarized in the number of *distinguishable distributions* (models), as illustrated in Figure 1. In general, useful information is closely related to *distinguishability*. In summary, if there are $C_n(\Theta)$ such distinguishable distributions, it is natural to call $I_n(\Theta) = \log C_n(\Theta)$ the *useful information*.

Let us estimate $C_n(\Theta)$ in the MDL (Minimum Description Length) world, as discussed in [3, 28, 60]. As a distance between distributions/models we adopt the Kullback-Leibler (KL) divergence D. Using Taylor expansion around $\hat{\theta}$, we find

$$\begin{aligned} D(P_{\hat{\theta}} \,||\, P_\theta) &:= \mathbf{E}[\log P_{\hat{\theta}}(X^n)] - \mathbf{E}[\log P_\theta(X^n)] \\ &= \frac{1}{2}(\theta-\hat{\theta})^T I(\hat{\theta})(\theta-\hat{\theta}) + o(\|\theta-\hat{\theta}\|^2), \end{aligned} \tag{1}$$

where $I(\theta) = \{I_{ij}(\theta)\}_{ij}$ is the *Fisher information matrix* defined as

$$I_{ij}(\theta) = -\mathbf{E}\left[\frac{\partial^2}{\partial\theta_i \partial\theta_j} \log P_\theta(X)\right].$$

As a distance we use

$$d_I(\theta,\theta_0) = \sqrt{(\theta-\hat{\theta})^T I(\hat{\theta})(\theta-\hat{\theta})}$$

which is the so called *Mahalanobis distance* [28]. This is a rescaled version of Euclidean distance, and by (1) we have $d_I(\theta,\theta_0) = O(\sqrt{D(\theta\|\theta_0)})$. One property of the d_I distance is that the volume V of a ball (ellipsoid) at center θ and radius ε is

$$V(B_I(\theta,\varepsilon)) = 1\Big/\sqrt{\det I(\theta)}\, V(B(\varepsilon),$$

where $B(\varepsilon)$ is the regular Euclidean ball and $\det I(\theta)$ is the determinant of $I(\theta)$ [3, 28].

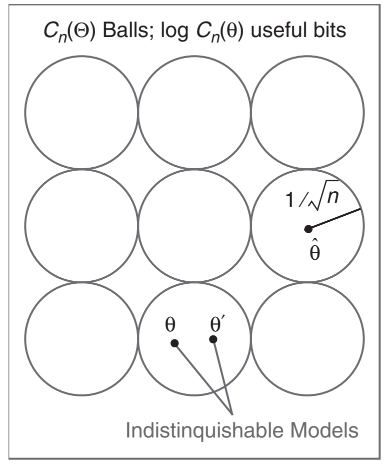

Fig. 1. Illustration to C(Θ).

To proceed, we need to specify the error probability and distinguishability. Let $B_{KL}(\theta_0,\varepsilon) = \{\theta : D(\theta\|\theta_0) \le \varepsilon\}$ be the KL-ball or radius ε around θ_0. Observe that the KL-ball $B_{KL}(\theta,\varepsilon)$ becomes $B_I(\theta,\sqrt{\varepsilon})$ ball in the d_I distance. The distinguishability of models depends on the error probability that can be estimated as follows [3] for some $\theta \in \Theta_0$ with $\dim(\Theta_0) = k$ [28]

$$\begin{aligned} P_\theta(\hat{\theta} \ne \theta) &= P_\theta(\arg\min_{\theta\in\Theta_0} D(\hat{\theta}(X^n)\,\|\,\theta) \ne \theta) \\ &\approx P_\theta(\theta(X) \notin B_{KL}(\theta,\varepsilon/n)) \sim 1 - O(\varepsilon^{k/2}) \end{aligned}$$

for some small $\varepsilon > 0$, where we use the fact that for Markov sources (more generally, for an exponential family of distributions)

$$\log\frac{P_{\hat{\theta}}(x^n)}{P_\theta(x^n)} = n\mathbf{E}_{\hat{\theta}}\left[\log\frac{P_{\hat{\theta}}(X)}{P_\theta(X)}\right] = nD(\hat{\theta}\,\|\,\theta).$$

We conclude that the number of distinguishable distributions $C_n(\Theta)$ is approximately equal to the *volume* $V_I(\Theta)$ of Θ under distance d_I divided by the volume of the ball size $B_I(\theta,\sqrt{\varepsilon/n})$. In [3] it is proved that

$$V_I(\Theta) = \int_\Theta \sqrt{\det I(\theta)}\, d\theta, \quad V(B_I(\theta,\sqrt{\varepsilon}) \approx O(\varepsilon^{k/2}/\sqrt{\det I(\theta)}).$$

Setting up the error probability at level $O(1/\sqrt{n})$ as indicated above, we conclude that the number of distinguishable distributions $C_n(\Theta)$ (i.e., the number of centers of the balls $B_I(\theta,\sqrt{\varepsilon})$) is (see [21, 28, 60])

$$C_n(\Theta) = \left(\frac{n}{2\pi}\right)^{k/2} \int_\Theta \sqrt{\det I(\theta)}\, d\theta + O(1) = \sum_{x^n} \sup_{\theta\in\Theta} P_\theta(x^n) = \inf_{\theta\in\Theta} \max_{x^n} \log\frac{P_{\hat{\theta}}}{P_\theta} \tag{2}$$

where the second equality follows from [28, 59]. In order to justify the last equality we need to turn our attention to the *maximal minimax redundancy*.

Let us begin with a precise information-theoretic definition of the minimax redundancy and its Shtarkov's bounds. Throughout this section, we write $L(C_n, x^n)$ for the length of a fixed-to-variable code $C_n; \mathcal{A}^n \to \{0,1\}^*$ assigned to the source sequence x^n over the alphabet $\mathcal{A} = \{1, 2, \ldots, m\}$ of size m that can be finite or not. In practice, one can only hope to have some knowledge about a *family* of sources $\mathcal{S}$ that generates the data, such as the family of memoryless sources $\mathcal{M}_0$ or Markov sources $\mathcal{M}_r$ of order $r > 0$. Following Davisson [18] and Shtarkov [69], we define the minimax worst-case (maximal) redundancy $R_n^*(\mathcal{S})$ for a family $\mathcal{S}$ as

$$R_n^*(\mathcal{S}) = \min_{C_n} \sup_{P \in \mathcal{S}} \min_{x_1^n} [L(C_n, x_1^n) + \log P(x_1^n)], \tag{3}$$

where C_n represents a set of prefix codes, and the source $P \in \mathcal{S}$ generates the sequence $x^n = x_1 \ldots x_n$. If we ignore the integer nature of the code length $L(C_n, x^n)$, then we can approximate it by $\log 1/P_\theta$ for some θ. Furthermore, $\log \sup_{P \in \mathcal{S}} P(x^n) = \log(1/P_{\hat\theta})$, where $\hat\theta$ is the ML estimator, so that

$$R_n^*(\mathcal{S}) = \inf_\theta \max_{x^n} \log \frac{P_{\hat\theta}}{P_\theta} + O(1) \tag{4}$$

which is the right-hand side of (2), and therefore $C(\Theta) = R_n^*(\mathcal{S}) + O(1)$.

We still need to justify the last equality in (2). We derive now Shtarkov's bound [69]. Define first the maximum likelihood distribution

$$Q^*(x^n) := \frac{\sup_{P \in \mathcal{S}} P(x^n)}{\sum_{y^n \in \mathcal{A}^n} \sup_{P \in \mathcal{S}} P(y^n)}.$$

Then observe [21]

$$\begin{aligned} R_n^*(\mathcal{S}) &= \min_{C_n} \sup_{P \in \mathcal{S}} \max_{x^n} (L(C_n, x^n) + \log P(x^n)) \\ &= \min_{C_n} \max_{x^n} \left(L(C_n, x^n) + \sup_{P \in \mathcal{S}} \log P(x^n)\right) \\ &= \min_{C_n} \max_{x^n} (L(C_n, x^n) + \log Q^*(x^n)) + \log \sum_{y^n \in \mathcal{A}^n} \sup_{P \in \mathcal{S}} P(y^n) \\ &= R_n^{GS}(Q^*) + \log \sum_{y^n \in \mathcal{A}^n} \sup_{P \in \mathcal{S}} P(y^n) = \log \sum_{y^n \in \mathcal{A}^n} \sup_{P \in \mathcal{S}} P(y^n) + O(1) \end{aligned}$$

where $0 < R_n^{GS}(Q^*) \le 1$ is the redundancy of the optimal generalized Shannon code (see [21]). Therefore, ignoring again the integer constraint (i.e., setting $R_n^{GS}(Q^*) = 0$) and using (4) rather than (3) we arrive at

$$\sum_{x^n} \sup_{\theta \in \Theta} P_\theta(x^n) = \inf_{\theta \in \Theta} \max_{x^n} \log \frac{P_{\hat\theta}}{P_\theta} = R_n^*(\mathcal{S})$$

which establishes the right-hand side of (2). From now on, we assume that $R^*(\mathcal{S}) = \log D_{n,m}(\mathcal{S})$ where

$$D_{n,m}(\mathcal{S}) = \sum_{x^n \in \mathcal{A}^n} \sup_{P \in \mathcal{S}} P(x^n). \tag{5}$$

The $O(1)$ term in (4) can be computed for finitely parameterized sources as in [21], but we will not elaborate on it here.

In summary, useful or learnable information is closely related to the minimax redundancy $R_n^*(\mathcal{S})$ which can be viewed as a measure of certain regularity properties of a source (regularity beyond the randomness/complexity expressed by the entropy). Next, using analytic tools we estimate asymptotically the minimax redundancy for various classes of sources such as memoryless for finite and infinite alphabets, renewal sources, and Markov sources. When discussing Markov sources, we rather turn our attention to combinatorial aspects of Markov types.

2.2. Minimax Redundancy for Memoryless Sources

In this section we study the minimax redundancy for a class of memoryless sources over finite and infinite alphabet of size m. We follow here [74]. Observe that $D_{n,m} := D_{n,m}(\mathcal{M}_0)$ defined in (5) takes the form

$$D_{n,m} = \sum_{k_1 + \cdots + k_m = n} \binom{n}{k_1, \ldots, k_m} \left(\frac{k_1}{n}\right)^{k_1} \cdots \left(\frac{k_m}{n}\right)^{k_m}, \tag{6}$$

where k_i is the number of times symbol $i \in \mathcal{A}$ occurs in a string of length n. Indeed, observing that $P(x^n) = p_1^{k_1} \cdots p_m^{k_m}$ where p_i are *unknown* parameters θ representing the probability for symbol $i \in \mathcal{A}$, we proceed as follows

$$\begin{aligned} D_n(\mathcal{M}_0) &= \sum_{x_1^n} \sup_{P(x_1^n)} P(x_1^n) \\ &= \sum_{x_1^n} \sup_{p_1, \ldots, p_m} p_1^{k_1} \cdots p_m^{k_m} \\ &= \sum_{k_1 + \cdots + k_m = n} \binom{n}{k_1, \ldots, k_m} \sup_{p_1, \ldots, p_m} p_1^{k_1} \cdots p_m^{k_m} \\ &= \sum_{k_1 + \cdots + k_m = n} \binom{n}{k_1, \ldots, k_m} \left(\frac{k_1}{n}\right)^{k_1} \cdots \left(\frac{k_m}{n}\right)^{k_m}, \end{aligned}$$

where the last line follows from

$$\sup_{P(x_1^n)} P(x_1^n) = \sup_{p_1, \ldots, p_m} p_1^{k_1} \cdots p_m^{k_m} = \left(\frac{k_1}{n}\right)^{k_1} \cdots \left(\frac{k_m}{n}\right)^{k_m}.$$

We should point out that (6) has a form that re-appears in the redundancy analysis of other sources. Indeed, the summation is over tuples $\mathbf{k} = (k_1, \ldots, k_m)$ representing a (memoryless) *type* (cf. Section 2.4) and under the sum the first term $\binom{n}{k_1, \ldots, k_m}$ counts the number of sequences x^n of the same type while the second term is the maximum likelihood distribution.

It is argued in [74] that the asymptotics of such a sum can be analyzed through its so-called *tree-like generating function* defined as

$$D_m(z) = \sum_{n=0}^\infty \frac{n^n}{n!} D_{n,m} z^n.$$

Here, we will follow the same methodology and employ the convolution formula for tree-like generating functions (cf. [76]). Observe that $D_m(z)$ relates to another tree-like generating function defined as

$$B(z) = \sum_{k=0}^\infty \frac{k^k}{k!} z^k.$$

This function, in turn, can be shown to be (cf. [76]) $B(z) = (1 - T(z))^{-1}$ for $|z| < e^{-1}$, where $T(z) = \sum_{k=1}^\infty (k^{k-1}/k!) z^k$ is the well-known *tree function*—that counts the number of rooted labeled trees on n vertices [24]—satisfying the implicit equation

$$T(z) = z e^{T(z)} \tag{7}$$

with $|T(z)| < 1$. The convolution formula [76] applied to (6) yields

$$D_m(z) = [B(z)]^m - 1. \tag{8}$$

Consequently, $D_{n,m} = (n!/n^n)[z^n][B(z)]^m$ where $[z^n]f(z)$ denotes the coefficient of z^n in $f(z)$.

Defining $\beta(z) = B(z/e)$, $|z| < 1$, noticing that $[z^n]\beta(z) = e^{-n}[z^n]B(z)$, and applying Stirling's formula, (8) yields

$$D_{n,m} = \sqrt{2\pi n}(1 + O(n^{-1}))[z^n][\beta(z)]^m. \tag{9}$$

Thus, it suffices to extract asymptotics of the coefficient at z^n of $[\beta(z)]^m$, for which a standard tool is Cauchy;s coefficient formula [24, 76], that is,

$$[z^n][\beta(z)]^m = \frac{1}{2\pi i}\oint \frac{\beta^m(z)}{z^{n+1}}dz \tag{10}$$

where the integration is around a closed path containing $z = 0$ inside which $\beta^m(z)$ is analytic. However, asymptotic evaluation of the above depends whether m is finite or is a function of n. We consider these two cases next.

2.2.1. Finite Alphabet Size

First we assume that the size of the alphabet m is finite and does not depend on n. This case was analyzed in [74] (see also [81, 82]). To evaluate the integral in (10) we apply Flajolet and Odlyzko *singularity analysis* [24, 76] because $[\beta(z)]^m$ has only algebraic singularities. Indeed, using (7) it can be shown that the singular expansion of $\beta(z)$ around its singularity $z = 1$ is [12]

$$\beta(z) = \frac{1}{\sqrt{2(1-z)}} + \frac{1}{3} - \frac{\sqrt{2}}{24}\sqrt{(1-z)} + O(1-z).$$

From [24, 76] we know that

$$[z^n](1-z)^{-\alpha} \sim \frac{n^{\alpha-1}}{\Gamma(\alpha)}, \quad \alpha \notin \{0, -1, -2, \ldots\}.$$

This is illustrated in Figure 2. The singularity analysis then yields the minimax redundancy [74]

$$R^*_{n,m} := \log D_{n,m} = \frac{m-1}{2}\log\left(\frac{n}{2}\right) + \log\left(\frac{\sqrt{\pi}}{\Gamma\left(\frac{m}{2}\right)}\right) + \frac{\Gamma\left(\frac{m}{2}\right)m\log e}{3\Gamma\left(\frac{m}{2}-\frac{1}{2}\right)}\cdot\frac{\sqrt{2}}{\sqrt{n}} + O\left(\frac{1}{n}\right) \tag{11}$$

for large n and fixed m, where Γ is the Euler gamma function. We conclude that the first term above coincides with Rissanen's lower bound: we pay a penalty of $\log n/2$ per unknown parameter.

2.2.2. Unbounded Alphabet

Now we assume that the alphabet size is unknown and *unbounded*. In fact, it may depend on n. When m grows with n, the singularity analysis *does not* apply because $\beta^m(z)$ grows exponentially with n. The growth of $\beta^m(z)$ determines that the *saddle point method* [24, 76], which we briefly review next, can be applied to (10). We will restrict our attention to a special case of the method, where the goal is to obtain an asymptotic approximation of

$$D_{n,m} = \sqrt{2\pi n}\frac{1}{2\pi i}\oint\frac{\beta(z)^m}{z^{n+1}}dz = \sqrt{2\pi n}\frac{1}{2\pi i}\oint e^{g(z)}dz,$$

where $g(z) = m\ln\beta(z) - (n+1)\ln z$. For example, when $m = n+1$ the function under the integral grows as $\exp((n+1)[\ln\beta(z) - \ln z])$ which becomes very large around z where $\ln\beta(z) - \ln z$ is maximized, and almost negligible everywhere else. This determines the asymptotics.

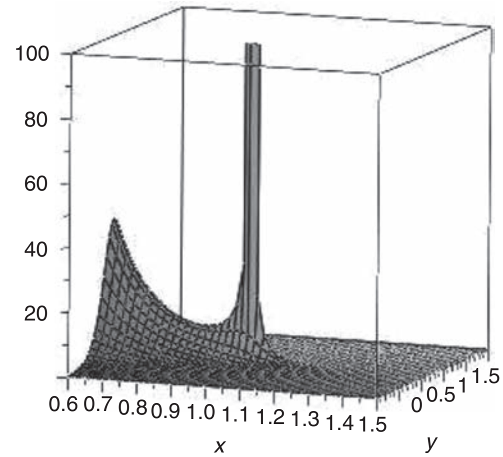

Fig. 2. Singularity analysis.

In general for any m and n, the saddle point z_0 is a solution of $g'(z_0) = 0$, which yields

$$g(z) = g(z_0) + \frac{1}{2}(z-z_0)^2 g''(z_0) + O(g'''(z_0)(z-z_0)^3).$$

Under mild conditions (see Table 8.4 in [76]), satisfied by our $g(z)$ (e.g., z_0 is real and unique), the saddle point method leads to

$$D_{n,m} = \sqrt{2\pi n}\frac{e^{g(z_0)}}{\sqrt{2\pi|g''(z_0)|}}\times\left(1 + O\left(\frac{g'''(z_0)}{(g''(z_0))^\rho}\right)\right),$$

for some $\rho < 3/2$. In our case, the saddle point z_0 varies from near 1 to near 0 depending on the relation between n and m as illustrated in Figure 3. It turns out that three cases must be considered: $m = o(n)$ (the saddle point $z_0 \approx 1$), $m = O(n)$(saddle point $0 < z_0 < 1$), and the case $n = o(m)$ (in this case $z_0 \approx 0$).

The following result is proved in [78] (see also [56])

Theorem 1 (Szpankowski and Weinberger, 2012). *For memoryless sources $\mathcal{M}_0$ over an m-ary alphabet, where $m \to \infty$ as n grows, the minimax worst-case redundancy behaves asymptotically as follows:*

(i) *For* $m = o(n)$

$$R^*_{n,m} = \frac{m-1}{2}\log\frac{n}{m} + \frac{m}{2}\log e + \frac{m\log e}{3}\sqrt{\frac{m}{n}} - \frac{1}{2} - \frac{\log e}{4}\sqrt{\frac{m}{n}} + O\left(\frac{m^2}{n} + \frac{1}{\sqrt{m}}\right). \tag{12}$$

(ii) *For* $m = \alpha n + \ell(n)$, *where* α *is a positive constant and* $\ell(n) = o(n)$,

$$R^*_{n,m} = n\log B_\alpha + \ell(n)\log C_\alpha - \log\sqrt{A_\alpha} - \frac{\ell(n)^2\log e}{2n\alpha^2 A_\alpha} + O\left(\frac{\ell(n)^3}{n^2} + \frac{\ell(n)}{n} + \frac{1}{\sqrt{n}}\right), \tag{13}$$

where

$$C_\alpha := \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4}{\alpha}}, \quad A_\alpha := C_\alpha + \frac{2}{\alpha}, \quad B_\alpha = \alpha C_\alpha^{\alpha+2}e^{-\frac{1}{C_\alpha}}.$$

(iii) *For* $n = o(m)$

$$R^*_{n,m} = n\log\frac{m}{n} + \frac{3}{2}\frac{n^2}{m}\log e - \frac{3}{2}\frac{n}{m}\log e + O\left(\frac{1}{\sqrt{n}} + \frac{n^3}{m^2}\right). \tag{14}$$

In summary, we conclude that for finite m and $m = o(n)$ the minimax redundancy, representing useful information

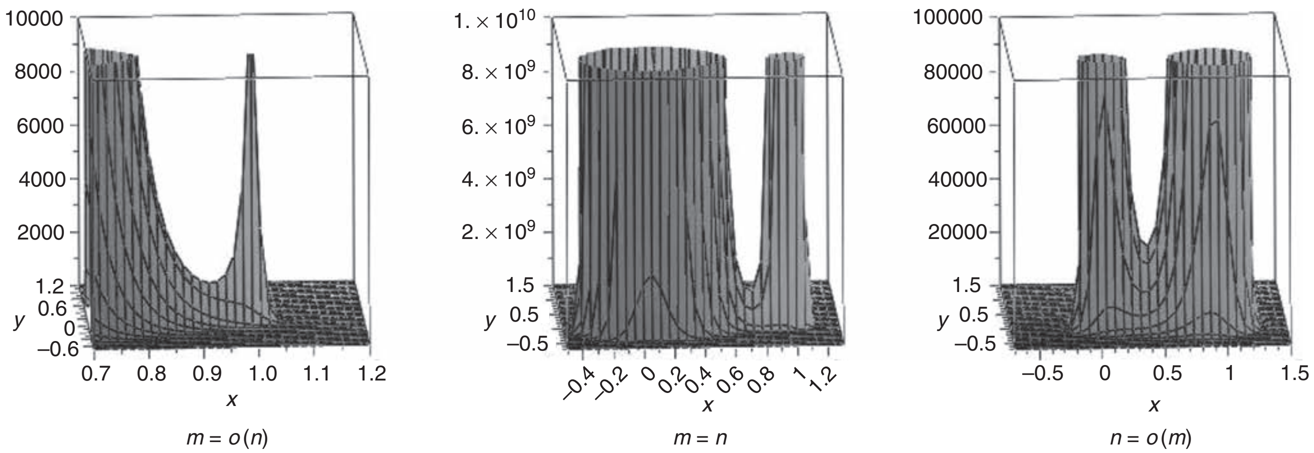

Fig. 3. Illustration of the saddle point method for Theorem 1.

embodied in regularity properties of a sequence, grows like $(m-1)/2 \times \log(n/m)$. This coincides with Rissanen's lower bound. However, for $m = O(n)$ the minimax redundancy grows linearly with n while for m growing faster than n the growth of the minimax redundancy is $n\log(m/n)$.

2.3. Minimax Redundancy for Renewal Sources

Let us continue our analytic *extravaganza* and consider non-finitely parameterized sources before we return to Markovian sources over finite alphabets in the next section. We study here the so called *renewal sources* first introduced in 1996 by Csiszár and Shields [15]. Such a source is defined as follows:

- Let $T_1, T_2 \dots$ be a sequence of i.i.d. positive-valued random variables with distribution $Q(j) = \Pr\{T_i = j\}$.
- The process $T_0, T_0 + T_1, T_0 + T_1 + T_2, \dots$ is a renewal process.
- In a binary renewal sequence the positions of the 1's are at the renewal epochs $T_0, T_0 + T_1, \dots$ with runs of zeros of lengths $T_1 - 1$, $T_2 - 1$, … in between the 1's.
- The process starts with $x_0 = 1$.

We follow here the analysis presented in [23]. A sequence generated by such a source becomes

$$x_0^n = 10^{\alpha_1}10^{\alpha_2}1\cdots10^{\alpha_n}1\underbrace{0\cdots0}_{k^*}$$

where k_m is the number of i such that $\alpha_i = m$. Then

$$P(x_1^n) = [Q(0)]^{k_0}[Q(1)]^{k_1}\cdots[Q(n-1)]^{k_{n-1}}\Pr\{T_1 > k^*\}.$$

The last term introduces some difficulties in finding the maximum likelihood distribution, but it can be proved that the minimax redundancy $R_n^*(\mathcal{R}_0) = \log D_n(\mathcal{R}_0)$ of the renewal source $\mathcal{R}_0$ satisfies

$$r_{n+1} - 1 \le D_n(\mathcal{R}_0) \le \sum_{m=0}^{n} r_m$$

where $r_n = \sum_{k=0}^{n} r_{n,k}$ and

$$r_{n,k} = \sum_{\mathcal{I}(n,k)} \binom{k}{k_0 \cdots k_{n-1}} \left(\frac{k_0}{k}\right)^{k_0}\left(\frac{k_1}{k}\right)^{k_1}\cdots\left(\frac{k_{n-1}}{k}\right)^{k_{n-1}}. \qquad (15)$$

Above $\mathcal{I}(n,k)$ is is the integer partition of n into k terms, i.e.,

$$n = 1k_0 + 2k_1 + \cdots + nk_{n-1}, \quad k = k_0 + \cdots + k_{n-1}.$$

Since r_n is too difficult to analyze, we rather study $s_n = \sum_{k=0}^{n} s_{n,k}$ where

$$s_{n,k} = e^{-k}\sum_{\mathcal{P}(n,k)} \frac{k^{k_0}}{k_0!}\cdots\frac{k^{k_{n-1}}}{k_{n-1}!}, \qquad \frac{r_{n,k}}{s_{n,k}} = \frac{k!}{k^k e^{-k}}$$

since

$$S(z,u) = \sum_{k,n} s_{n,k}(u/e)^k z^n = \sum_{\mathcal{P}_{n,k}} z^{1k_0+2k_1+\cdots+nk_{n-1}}\left(\frac{u}{e}\right)^{k_0+\cdots+k_{n-1}}\frac{k^{k_0}}{k_0!}\cdots\frac{k^{k_{n-1}}}{k_{n-1}!}$$
$$= \prod_{i=1}^{\infty}\beta(z^i u)$$

where $\beta(z) = B(z/e)$ is defined in the previous section.

To compare s_n to r_n, we introduce the random variable K_n as follows

$$\Pr\{K_n = k\} = \frac{s_{n,k}}{s_n}.$$

Stirling's formula yields

$$\frac{r_n}{s_n} = \sum_{k=0}^{n}\frac{r_{n,k}}{s_{n,k}}\frac{s_{n,k}}{s_n} = \mathbf{E}[(K_n)!K_n^{-K_n}e^{K_n}]$$
$$= \mathbf{E}[\sqrt{2\pi K_n}] + O(\mathbf{E}[K_n^{-\frac{1}{2}}]).$$

Thus

$$r_n = s_n\mathbf{E}[\sqrt{2\pi K_n}](1+o(1)) = s_n\sqrt{2\pi\mathbf{E}[K_n]}\,(1+o(1)).$$

To understand probabilistic behavior of K_n, we apply sophisticated tools of analytic combinatorics such as Mellin transform and the saddle point [24, 76]. In particular, we must evaluate $[z^n]S(z,1)$ by the saddle point that leads to the following

$$s_n = [z^n]S(z,1) = [z^n]\exp\left(\frac{c}{1-z} + a\log\frac{1}{1-z}\right)$$

which is illustrated in Figure 4. We prove in [23] the following.

Lemma 1. *Let* $\mu_n = \mathbf{E}[K_n]$ *and* $\sigma_n^2 = \mathbf{Var}(K_n)$*. Then*

$$\mu_n = \frac{1}{4}\sqrt{\frac{n}{c}}\log\frac{n}{c} + o(\sqrt{n}), \qquad \sigma_n^2 = O(n\log n) = o(\mu_n^2),$$

where $c = \pi^2/6 - 1$, $d = -\log 2 - \frac{3}{8}\log c - \frac{3}{4}\log \pi$.

This leads to our final result proved in [23].

Theorem 2 (Flajolet and Szpankowski, 1998). *We have the following asymptotics*

$$s_n \sim \exp\left(2\sqrt{cn} - \frac{7}{8}\log n + O(1)\right),$$

$$\log r_n = \frac{2}{\log 2}\sqrt{cn} - \frac{5}{8}\log n + \frac{1}{2}\log\log n + O(1).$$

that yields

$$R_n^*(\mathcal{R}_0) = \frac{2}{\log 2}\sqrt{cn} + O(\log n).$$

where $c = \frac{\pi^2}{6} - 1 \approx 0.645$.

In passing we should point out that the renewal source technically is reminiscent of the memoryless sources with unbounded alphabet (cf (3) and (15)). The analysis of renewal sources is, however, much more sophisticated.

2.4. Markov Minimax Redundancy and Markov Types

In this section, we return to the finite size alphabet $\mathcal{A} = \{1, \ldots, m\}$ but now we consider a class $\mathcal{M}_1$ of Markovian sources of order $r = 1$. More precisely, the probability of a sequence x^n is given by

$$P(x^n) = P(x_1)\prod_{i,j=1}^{m} p_{ij}^{k_{ij}}$$

where k_{ij} is the number of pairs $(i,j) \in \mathcal{A}^2$ in x^n, p_{ij} are the (unknown) transition probabilities while $P(x_1)$ is the initial probability. Then the minimax redundancy (ignoring again the integer nature of coding) is [37]

$$D_n(\mathcal{M}_1) = \sum_{x_1^n}\sup_P P(x^n) = \sum_{k\in Q_n(m)} |\mathcal{T}_n(\mathbf{k})| \left(\frac{k_{11}}{k_1}\right)^{k_{11}} \cdots \left(\frac{k_{mm}}{k_m}\right)^{k_{mm}}, \quad (16)$$

where $Q_n(m)$ denotes a set of *Markov types* discussed in the sequel, and $T_n(\mathbf{k}) := \mathcal{T}_n(x_1^n)$ is the number of sequences of the same Markov type represented by the frequency count matrix $\mathbf{k} = \{k_{ij}\}_{i,j\in\mathcal{A}^2}$. The *frequency matrix* $\mathbf{k}$, which we also write $[k_{ij}]$, satisfies two important properties

$$\sum_{i,j\in\mathcal{A}} k_{ij} = n - 1, \quad (17)$$

and additionally for any $i \in \mathcal{A}$ [37, 83]

$$\sum_{j=1}^{m} k_{ij} = \sum_{j=1}^{m} k_{ji} + \delta(x_1 = i) - \delta(x_n = i), \quad \forall i \in \mathcal{A}, \quad (18)$$

where $\delta(A) = 1$ when A is true and zero otherwise. The last property is called the *flow conservation property* and is a consequence of the fact that the number of pairs starting with symbols $i \in \mathcal{A}$ must be equal to the number of pairs ending with symbol $i \in \mathcal{A}$

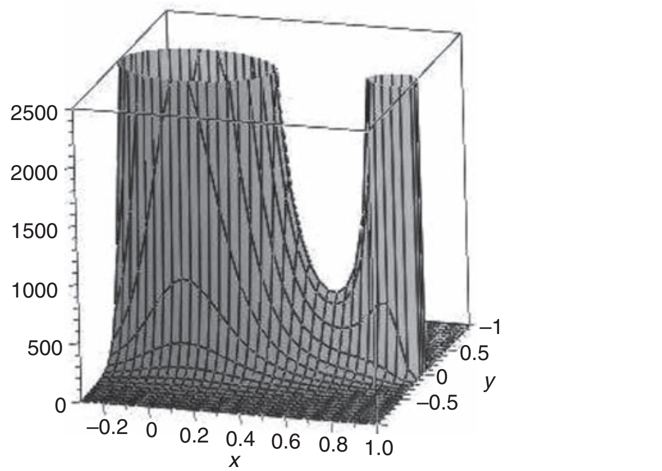

Fig. 4. Saddle Point.

with the possible exception of the first and last pairs. To avoid this exception, hereafter we focus on *cyclic* strings in which the first element x_1 follows the last x_n. For such cyclic strings the frequency matrix $\mathbf{k}$ satisfies a simplified system of linear equations, namely

$$\sum_{i,j\in\mathcal{A}} k_{ij} = n, \quad (19)$$

$$\sum_{j=1}^{m} k_{ij} = \sum_{j=1}^{m} k_{ji}, \quad \forall\, i \in \mathcal{A}. \quad (20)$$

Such integer matrices $\mathbf{k}$ will be called *balanced frequency matrices* or simply balanced matrices. We also call (20) the "conservation law" equation or simply the *balanced boundary condition* (BBC). We denote by $\mathcal{F}_n(m)$ the set of nonnegative integer solutions of (19) and (20).

We are now ready to define cyclic Markov types. Two cyclic sequences have the same (cyclic) Markov type if they have the same empirical distribution

$$P(x^n) = \prod_{i,j\in\mathcal{A}} p_{ij}^{k_{ij}}.$$

Thus, we assume the initial condition is a cyclic one. We denote by $\mathcal{P}_n(m)$ the set of cyclic Markov types and enumerate them by comparing them to the cardinality of $\mathcal{F}_n(m)$, and also to the set of Markov types $\mathcal{Q}_n(m)$ over linear strings. In passing, we should point out that for a given sequence x^n, the *type class* is defined as

$$\mathcal{T}_n(x^n) = \{y^n : P(x^n) = P(y^n)\}$$

for all empirical distributions P_{x^n} in a given model class. Clearly, $\bigcup_{x^n}\mathcal{T}_n(x^n) = \mathcal{A}^n$, and $|\mathcal{T}_n(x^n)|$ counts the number of sequences of the same type as x^n; it is required to estimate the minimax redundancy for Markov sources as shown in (16).

Our goal is to enumerate the number of cyclic Markov types $|\mathcal{P}_n(m)|$ that from now on we simply call Markov types. Enter combinatorics: we shall show that the number of Markov types is asymptotically equivalent to estimating: (i) the number of the balanced frequency matrices, (ii) the number of integer solutions $|\mathcal{F}_n(m)|$ of a system of *linear Diophantine equations* (19)–(20), and finally (iii) the number of connected Eulerian multigraphs, as defined next. To see the latter, we present another characterization of

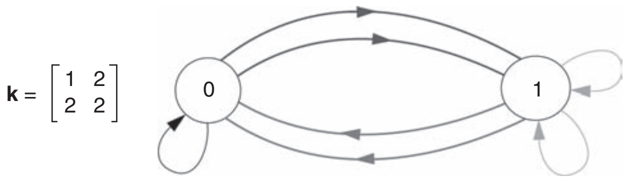

Fig. 5. A frequency matrix and its corresponding Eulerian graph.

Markov types. Let us define a directed multigraph $G = (V, E)$ with the set of vertices $V = \mathcal{A}$ and k_{ij} edges between vertices $i, j \in \mathcal{A}$ For $\mathcal{A} = \{0, 1\}$ such a graph is shown in Figure 5. Then, as already observed in [6, 27, 37], the number of sequences of a given type $\mathbf{k}$, i.e., $|\mathcal{T}(\mathbf{k})|$, is equal to the number of Eulerian cycles in G. On the other hand, the number of types $|\mathcal{P}_n(m)|$ coincides with the number of Eulerian digraphs $G = (V, E)$ such that $V \subseteq \mathcal{A}$ and $|E| = n$ (here $V \subseteq \mathcal{A}$ since there may be sequences composed of only *some* symbols of the alphabet). The point we emphasize is that G may be defined over a *subset* of $\mathcal{A}$, as shown in the next Figure 6 (i.e., there may be some isolated vertices).

Let us explore further these two sets $\mathcal{P}_n(m)$ and $\mathcal{F}_n(m)$ in the language of graphs. In fact, we need to introduce another set. We denote it by $\mathcal{E}_n(m)$, the set of connected Eulerian digraphs on $\mathcal{A}$; the middle of Figure 6 shows an example of a graph in this set. Finally, the set $\mathcal{F}_n(m)$ can be viewed as the set of digraphs G with $V(G) = \mathcal{A}$, $|E(G)| = n$ and satisfying the flow conversation property (in-degree equals out-degree). We call such graphs *conservative digraphs*. Observe that a graph in $\mathcal{F}_n(m)$ may consist of several connected (not communicating) Eulerian digraphs, as shown in the third example in Figure 6.

There is a simple relation between $|\mathcal{E}_n(m)|$ and $|\mathcal{P}_n(m)|$. Indeed,

$$|\mathcal{P}_n(m)| = \sum_k \binom{m}{k} |\mathcal{E}_n(k)| \tag{21}$$

since there are $\binom{m}{k}$ ways to choose $m - k$ isolated vertices in $\mathcal{P}_n(m)$. Now, observe that a conservative digraph may have several connected components. Each connected component is either a connected Eulerian digraph or an isolated node without an edge. This leads to

$$|\mathcal{F}_n(m)| = |\mathcal{E}_n(m)| + \sum_{i=2}^{m} \sum_{\mathcal{A} = \mathcal{A}_1 \cup \cdots \cup \mathcal{A}_i} \sum_{n_1 + \cdots + n_j = n} \prod_{j=1}^{i} |\mathcal{E}_{n_j}(\mathcal{A}_j)| \tag{22}$$

where the sum is over all (unordered) set partitions $\mathcal{A} = \mathcal{A}_1 \cup \cdots \cup \mathcal{A}_i$ into $i \geq 2$ (nonempty) parts with n_j edges in each di-subgraph $\mathcal{E}_{n_j}(\mathcal{A}_j)$ over $\mathcal{A}_j$ vertices. Observe that every set partition $\mathcal{A} = \mathcal{A}_1 \cup \cdots \cup \mathcal{A}_i$ with $|\mathcal{A}_j| = m_j > 0$ is a partition of $\mathcal{A}$ into i *distinguished* subsets of cardinality m_j. In fact, using the so called *exponential formula* [24] (page 118) we may conclude even more, namely [34]

$$|\mathcal{F}_n(m)| = |\mathcal{E}_n(m)| + \sum_{i=2}^{m} \frac{1}{i!} \sum_{m_1 + \cdots + m_i = m} \binom{m}{m_1 \cdots m_i} \sum_{n_1 + \cdots n_i = n} \prod_{j=1}^{i} |\mathcal{E}_{n_j}(m_j)| \,.$$

A direct consequence of this is the following asymptotic equivalence [34].

Lemma 2. *The following holds for all $m \geq 2$ and $n \to \infty$*

$$|\mathcal{F}_n(m)| = |\mathcal{P}_n(m)| + O(2^m m^3 n^{m^2 - 3m + 3}). \tag{23}$$

In view of the above we need to enumerate the number of solutions $|\mathcal{F}_n(m)|$ of the system of linear Diophantine equations (19)–(20). Again, we accomplish it by analytic methods. Let

$$F_m^*(z) = \sum_{n \geq 0} |\mathcal{F}_n(m)| z^n.$$

However, to find $F_m^*(z)$ we need to evaluate a more complicated generating function that enumerates all balanced matrices, that is,

$$F_m^*(\mathbf{z}) = \sum_{\mathbf{k} \in \mathcal{F}_n(m)} \mathbf{z}^{\mathbf{k}},$$

where $\mathbf{z}^{\mathbf{k}} := \prod_{ij} z_{ij}^{k_{ij}}$. Notice that the summation is over all balances matrices $\mathbf{k} \in \mathcal{F}_n(m)$. This is a daunting task, but we can easily compute the above generating function if the summation is over *all* matrices (satisfying only (19)). Indeed,

$$F_m(\mathbf{z}) = \sum_{\mathbf{k}} \mathbf{z}^{\mathbf{k}} = \prod_{ij} (1 - z_{ij})^{-1}. \tag{24}$$

The remaining problem is to translate $F_m(\mathbf{z})$ into $F_m^*(\mathbf{z})$. This is presented in the next lemma, where we consider a multivariate generating functions $G(\mathbf{z}) = \sum_{\mathbf{k}} g_{\mathbf{k}} \mathbf{z}^{\mathbf{k}}$ and $G^*(\mathbf{z}) = \sum_{\mathbf{k} \in \mathcal{F}} g_{\mathbf{k}} \mathbf{z}^{\mathbf{k}} = \sum_{n \geq 0} \sum_{\mathbf{k} \in \mathcal{F}_n(m)} g_{\mathbf{k}} \mathbf{z}^{\mathbf{k}}$ over general sequences $g_{\mathbf{k}}$ indexed by matrices $\mathbf{k}$. The following was proved in [37].

Lemma 3. *Let $G(\mathbf{z}) = \sum_{\mathbf{k}} g_{\mathbf{k}} \mathbf{z}^{\mathbf{k}}$ be the generating function of a complex matrix $\mathbf{z}$. Then*

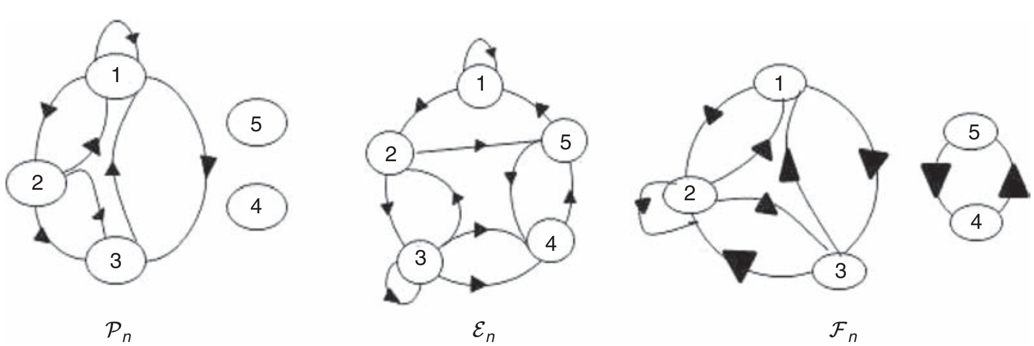

Fig. 6. Examples of graphs belonging to $\mathcal{P}_7(5)$, $\mathcal{E}_{11}(5)$ and $\mathcal{F}_9(5)$ sets.

$$G^*(\mathbf{z}) := \sum_{n\geq 0}\sum_{\mathbf{k}\in\mathcal{F}_n} g_{\mathbf{k}}\mathbf{z}^{\mathbf{k}} = \left(\frac{1}{2i\pi}\right)^m \oint \frac{dx_1}{x_1}\cdots\oint\frac{dx_m}{x_m} G\left(\left[z_{ij}\frac{x_j}{x_i}\right]\right)$$

with the convention that the ij-th coefficient of the matrix $[z_{ij}(x_j/x_i)]$ *is* $z_{ij}(x_j/x_i)$, *and* $\mathbf{i} = \sqrt{-1}$. *In other words,* $[z_{ij}(x_j/x_i)] = \Delta^{-1}(x)\mathbf{z}\Delta(x)$ *where* $\Delta(x) = \mathrm{diag}(x_1, \ldots, x_m)$.

Proof. Observe that

$$G(\Delta^{-1}(x)\mathbf{z}\Delta(x)) = G\left(\left[z_{ij}\frac{x_j}{x_i}\right]\right) = \sum_{\mathbf{k}} g_{\mathbf{k}}\mathbf{z}^{\mathbf{k}}\prod_{i=1}^m x_i^{\sum_j k_{ji}-\sum_j k_{ji}}. \tag{25}$$

Therefore, $G^*(\mathbf{z})$ is the coefficient of $G([z_{ij}(x_j/x_i)])(\cdot)$ at $x_1^0x_2^0\cdots x_m^0$ denoted as $[x_1^0\cdots x_m^0]$ since $\sum_j k_{ji} - \sum_j k_{ji} = 0$ for matrices $\mathbf{k}\in\mathcal{F}$. The result follows from the Cauchy coefficient formula (cf. [76]).

Now we are ready to enumerate $\mathcal{F}_n(m)$. Setting in Lemma 2 $z_{ij} = zx_i/x_j$ and using (24) we conclude that

$$F_m^*(z) = \frac{1}{(1-z)^m}[x_1^0x_2^0\cdots x_m^0]\prod_{i\neq j}\left[1 - z\frac{x_i}{x_j}\right]^{-1}. \tag{26}$$

Thus, by the Cauchy formula

$$|\mathcal{F}_n(m)| = [z^n]F_m^*(z) = \frac{1}{2\pi i}\oint\frac{F_m^*(z)}{z^{n+1}}dz.$$

This allows us to formulate our main result on the enumeration of (cyclic) Markov types.

Theorem 3 (Knessl, Jacquet, and Szpankowski, 2012). (i) Cyclic Types. *For fixed m and* $n\to\infty$ *the number of cyclic Markov types is*

$$|\mathcal{P}_n(m)| = d(m)\frac{n^{m^2-m}}{(m^2-m)!} + O(n^{m^2-m-1}) \tag{27}$$

where $d(m)$ *is a constant that also can be expressed by the following integral*

$$d(m) = \frac{1}{(2\pi)^{m-1}}\underbrace{\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}}_{(m-1)-fold}\prod_{j=1}^{m-1}\frac{1}{1+\varphi_j^2}$$

$$\times\prod_{k\neq \ell}\frac{1}{1+(\varphi_k-\varphi_\ell)^2}\,d\varphi_1 d\varphi_2\cdots d\varphi_{m-1}. \tag{28}$$

When $m\to\infty$ *we find that*

$$|\mathcal{P}_n(m)| \sim \frac{\sqrt{2}\,m^{3m/2}e^{m^2}}{m^{2m^2}2^m\pi^{m/2}}\cdot n^{m^2-m} \tag{29}$$

provided that $m^4 = o(n)$.

(ii) Markov Types. *The number of Markov types* $|\mathcal{Q}_n(m)|$ *with arbitrary initial conditions satisfies*

$$|\mathcal{Q}_n(m)| = (m^2-m+1)|\mathcal{P}_n(m)|(1 - O(n^{-2m}))$$

where $|\mathcal{P}_n(m)|$ *is presented in (i).*

In order to finish our analysis of the minimax redundancy, we need to estimate the number of sequences of a given type. First, we replace (16) by

$$D_n(\mathcal{M}_1) = m\sum_{b\in\mathcal{A}}\sum_{\mathbf{k}\in\mathcal{F}_n,\,k_{ba}>0}|\mathcal{T}_n^{ba}(k-[\delta_{ba}])|^{\mathbf{k}-[\delta_{ba}]}(k_b-1)^{-k_b+1}\prod_{i\neq b}(k_i)^{-k_i}, \tag{30}$$

where $|\mathcal{T}_n^{ba}(\mathbf{k})|$ is the number of cyclic strings x^n of type $\mathbf{k}$ starting with b and ending with a. To compute it we first introduce

$$B_{\mathbf{k}} = \binom{k_1}{k_{11}\cdots k_{1m}}\cdots\binom{k_m}{k_{m1}\cdots k_{mm}}$$

where $k_i = \sum_j k_{ij}$. It may be viewed as the number of ways to depart from all m vertices in the multiple graph G associated with the frequency matrix $\mathbf{k}$ (but not necessarily completing an Eulerian cycle). Observe that

$$B(\mathbf{z}) = \sum_{\mathbf{k}} B_{\mathbf{k}}\mathbf{z}^{\mathbf{k}} = \prod_{a\in\mathcal{A}}\left(1 - \sum_{b\in A} z_{a,b}\right)^{-1}.$$

Lemma 2 yields [37]

$$B^*(\mathbf{z}) = \sum_{\mathbf{k}\in\mathcal{F}_n(m)} B_{\mathbf{k}}\mathbf{z}^{\mathbf{k}} = \frac{1}{\det(\mathbf{I}-\mathbf{z})}$$

where $\mathbf{I}$ is the $m\times m$ identity matrix. Using this approach we can finally estimate the number of sequences starting with an a and finishing with a b of a given Markov type as follows

$$|\mathcal{T}_n^{ba}(\mathbf{k})| = \frac{k_{ba}}{k_b}B_{\mathbf{k}}\cdot\det_{bb}(\mathbf{I}-\mathbf{k}^*)\left(1+O\left(\frac{1}{n}\right)\right),$$

where $\mathbf{k}^*$ is the matrix whose ij-th element is k_{ij}/k_i, that is, $\mathbf{k}^* = [k_{ij}/k_i]$ (cf. [83, 37]).

Putting everything together, in [37] we prove the following asymptotic expansion for the minimax redundancy of Markov sources (cf. [59]).

Theorem 4 (Rissanen, 1996, Jacquet and Szpankowski, 2004). (i) *Let* $\mathcal{M}_1$ *be the class of Markov sources over a finite alphabet* $\mathcal{A}$ *of size m. The worst case minimax redundancy is* $R_n^*(\mathcal{M}_1) = \log D_n(\mathcal{M}_1)$ *where*

$$D_n(\mathcal{M}_1) = \left(\frac{n}{2\pi}\right)^{m(m-1)/2}A_m\times\left(1+O\left(\frac{1}{n}\right)\right) \tag{31}$$

with

$$A_m = m\int_{\mathcal{K}(1)}F_m(y_{ij})\prod_{i\in\mathcal{A}}\frac{\sqrt{\sum_{j\in\mathcal{A}}y_{ij}}}{\prod_{j\in\mathcal{A}}\sqrt{y_{ij}}}\prod_{ij\in\mathcal{A}^2}dy_{ij}$$

where

$$\mathcal{K}(1) = \left\{y_{ij}: y_{ij}\geq 0,\ \ \sum_{ij}y_{ij} = 1, \forall i: \sum_j y_{ij} = \sum_j y_{ji}\right\},$$

$F_m(\mathbf{y}) = \sum_{b\in\mathcal{A}}\det_{bb}(1-\mathbf{y}^*)$, *and* $\mathbf{y}^*$ *is the matrix whose ij-th coefficient is* $y_{ij}/\sum_{j'}y_{ij'}$.

(ii) *Let* $\mathcal{M}_r$ *be the class of Markov sources of order r over a finite alphabet* $\mathcal{A}$ *of size m. The minimax redundancy is* $R_n^*(\mathcal{M}_1) = \log D_n(\mathcal{M}_r)$ *where*

$$R_n^*(\mathcal{M}_r) = \left(\frac{n}{2\pi}\right)^{m^r(m-1)/2}A_m^r\times\left(1+O\left(\frac{1}{n}\right)\right) \tag{32}$$

with

$$A_m^r = m^r\int_{\mathcal{K}_r(1)}F_m^r(\mathbf{y})\prod_{w\in\mathcal{A}^r}\frac{\sqrt{y_w}}{\prod_j\sqrt{y_{w,j}}},$$

where $\mathcal{K}_r(1)$ *is the convex set of* $m^r\times m$ *matrices* $\mathbf{y}$ *with non-negative coefficients such that* $\sum_{w,j}y_{w,j} = 1$, $w\in\mathcal{A}^r$. *The function*

$$F_m^r(\mathbf{y}_r) = \sum_w \det_{ww}(\mathbf{I}-\mathbf{y}_r^*),$$

where $\mathbf{y}_r^*$ is the $m^r \times m^r$ matrix whose (w,w') coefficient is equal to $y_{w,a}/\sum_{i\in\mathcal{A}} y_{wi}$ if there exist a in $\mathcal{A}$ such that w' is a suffix of wa, otherwise the (w,w')th coefficient is equal to 0.

The evaluation of the constants A_m is not easy. But, for a binary alphabet $(m=2)$ we have

$$A_2 = 2\int_{\mathcal{K}(1)} (\det_{11}(\mathrm{I}-\mathbf{y}^*) + \det_{22}(\mathbf{I}-\mathbf{y}^*)) \times \frac{\sqrt{y_1}}{\sqrt{y_{11}}\sqrt{y_{12}}}\frac{\sqrt{y_2}}{\sqrt{y_{21}}\sqrt{y_{22}}}\,dy_{11}dy_{12}dy_{21}dy_{22}. \quad (33)$$

Since $\det_{11}(\mathbf{I}-\mathbf{y}^*) = (y_{21}/y_2)$ and $\det_{22}(\mathbf{I}-\mathbf{y}^*)$ by symmetry, and since the condition $\mathbf{y} \in \mathcal{K}(1)$ means $y_1 + y_2 = 1$ and $y_{12} = y_{21}$ we arrive at, $A_2 = 16 \cdot G$ where G is the Catalan constant defined as $G = \sum_i((-1)^i/(2i+1)^2) \approx 0.915965594$.

3. Science of Information: Beyond Shannon

In *science of information* the goal is to pursue the theory of information beyond Shannon's original objectives (of communication), by applying it to problems of biology, neuroscience, economics, physics, and massive data where knowledge extraction is the game changer. We believe that in order to make fundamental contributions to these applications, we first need better understanding of new aspects of temporal, spatial, structural and semantic information. In this section, we first briefly review some recent results on semantic and temporal properties and on cooperation, to focus on structural information.

3.1. Delay, Semantic, and Cooperation

The mathematical theory of information arose from Shannon's theorem on channel capacity, defined as the maximum rate that can be achieved over a channel with asymptotically small probability of error. Shannon capacity of a channel places no restrictions on complexity or delay in transmission or reception. Methods to properly characterize the complexity and the **delay** could potentially fill a large gap that would extend Shannon capacity to dynamic networks with multi-point communication and often unpredictable delays [26, 29]. Furthermore, the increasing demands for using wireless networks require such delay guarantees. Applications include VoIP, video streaming, real time surveillance, networked control, etc. One common characteristic of these applications is that they have a strict deadline associated with each packet. Further, the channel reliabilities of different clients can be different, and can even vary over time. These are compelling reasons why we need to understand the role of delay in distributed communication.

In [58] Polyanskiy, Poor, and Verdu extend the fundamental channel coding theorem of Shannon to a finite block-length regime. In particular, it is shown that coding rate $M_n^*(n,\varepsilon)$ for finite block length n is

$$\frac{1}{n}\log M^*(n,\varepsilon) \approx C - \sqrt{\frac{V}{n}}\,Q^{-1}(\varepsilon)$$

where C is the capacity, V is the channel dispersion, ε is error probability, and Q is the complementary Gaussian distribution. This is a non-asymptotic result (i.e., precise lower and upper bounds are presented), and it allows us to compute the degradation in capacity, even for small block lengths. Recently, these results are extended to lossy compression [47].

In another line of research in a real time coding system with lookahead, Asnani and Weissman [2] investigate the impact of delay on expected distortion. The system consists of a memoryless source; a memoryless channel; an encoder, which encodes the source symbols sequentially, with knowledge of future source symbols up to a fixed finite lookahead, with or without feedback of the past channel output symbols; and a decoder, which sequentially constructs the source symbols using the channel output. The objective is to minimize the expected per-symbol distortion using a control theory approach. The authors provide one of the first results in this line of research. This bridges the gap between causal encoding (delay = 0) and the infinite lookahead case (delay = ∞) where Shannon theoretic arguments show that encoding-decoding separation is optimal.

However, any further progress in information theory of networks requires us to understand distributed information and link delay with flow of information. In a novel line of research P.R. Kumar and co-authors [31, 42] design reliable scheduling policies with delay constraints for unreliable wireless networks. They focus on a formulation that appears to provide a useful and tractable framework for modeling, analyzing and designing real-time wireless communications. This framework is built on top of an analytical model that jointly considers the three important aforementioned challenges: a strict deadline for each packet, the timely throughput requirement specified by each client or application, and finally the unreliable and heterogeneous nature of wireless transmissions. An important feature is that this model is suitable for characterizing the needs of a wide range of applications, and the model allows each application to specify its individual demand.

We turn now our attention to **semantic** aspect of information. Shannon in his 1948 paper asserted "Frequently the messages have meaning, that is, they refer to or are correlated according to some system with certain physical or conceptual entities. These semantic aspects of communication are irrelevant to the engineering problem." However, Sudan and his collaborators [25, 40] argue that the meaning of information does start to become relevant whenever there is diversity in the communicating parties and when parties themselves evolve over time. For example, when a computer attempts to communicate with a printer they must talk the same language in the same format (i.e., "printer driver"). This leads Sudan and his collaborators to consider communication in the setting where encoder and decoder do not agree a priori on the communication protocols, thus encoder and decoder do not understand each other. In [25, 40] a mathematical theory of goal-oriented communication is proposed from the complexity theory point of view. Perhaps these are among the first results that may lead to a new information theory of semantic communication.

Finally, we discuss information theory of **cooperation and dependency.** In an extension of the Shannon framework, Cuff, Permuter and Cover [17] initiate a theory of cooperation and coordination in networks. A general understanding of the limits of dependence yields rate distortion theory (data compression) as a special case and provides a general approach to distributed data compression and cooperation. It also elucidates such diverse processes as intercellular biological communication. The role of dependence is exemplified by the telephone system, wireless communication, the internet, news services, the economies of large countries and the internal workings of computer architecture. The efficacy of all of these systems depends on fast communication and consequent cooperative behavior. Such distributed dependence is also found in chemical reactions, landslides, hurricanes, the dynamics of the sun and the universe itself. What are the necessary information exchanges? What limits on physical dependence are imposed by the speed of information? Are there energy constraints on computation? Some of these vast generalities can be addressed by developing a science of information for dependence. In [17] the authors ask what dependence can be established among nodes given communication constraints. More precisely, the authors compute the achievable joint distribution among network nodes, provided that the communication rates are given. Such a distributed cooperation can be the solution to many problems, such as distributed games, distributed control, and bounds on the influence of one part of a physical system on another.

Dependency and rational expectation are critical ingredients in Sims' work on modern dynamic economic theory [51]. Sims points out that existing theories of rational expectations with continuous optimization imply infinite mutual information between market and person actions. By imposing information flow constraints, discrete behavior emerges (as already seen in [61]) that better describe real economic behavior (cf. also [71]).

3.2. Information Content of Graphical Structures

Structural information appears in myriad applications, from biology to social networks to material sciences. In fact, in recent years we have become inundated with new (unconventional) data: the internet, social networks, biological networks, and medical records are all key examples that present grand challenges. For instance, in recent paper [80] Varshney *et al.* reported a pretty complete wiring (graph) of 302 neurons in the *C.elegans* worm that allows inference of biological functions from the neuronal network structure.

Unconventional data often are represented by more sophisticated data structures such as graphs, sets, and trees. For example, a graph can be described by a binary matrix that further can be viewed as a binary sequence. However, such a sequence does not exhibit internal symmetries that are conveyed by the so-called graph automorphism (such automorphisms make certain sequences/matrices "indistinguishable"). The main challenge in dealing with such structural data is to identify and describe these structural relations. In fact, these "regular properties" constitute "useful (extractable) information" discussed in Section 2.1. Furthermore, such data structures often have two types of information: the information conveyed by the structure itself, and the information conveyed by the data labels implanted in the structure. We still do not have good metrics of information embodied in structure.

As the first step in understanding structural information, we restrict our attention to structures on graphs, specifically, we study *unlabeled graphs* (or structures). In particular, given n distinguishable (labeled) vertices, a random graph is generated by adding edges randomly. This random graph model $\mathcal{G}$ produces a probability distribution on graphs, and the graph entropy $H_{\mathcal{G}}$ is defined naturally as

$$H_{\mathcal{G}} = \mathbf{E}[-\log P(G)] = -\sum_{G \in \mathcal{G}} P(G) \log P(G),$$

where $P(G)$ is the probability of a graph G. However, to focus on structural properties, we consider here unlabeled graphs in which the vertices are indistinguishable. We denote such an unlabeled graph by $S \in \mathcal{S}$ and clearly

$$P(S) = \sum_{G \cong S, G \in \mathcal{G}} P(G).$$

Here $G \cong S$ means that G and S have the same structure, that is, S is *isomorphic* to G. Thus, if all isomorphic labeled graphs have the same probability, then for any labeled graph $G \cong S$,

$$P(S) = N(S) \cdot P(G), \tag{34}$$

where $N(S)$ is the number of different labeled graphs that have the same structure as S. The *structural entropy* $H_{\mathcal{S}}$ of a random graph can be defined as the entropy of a random structure $\mathcal{S}$, that is,

$$Hs = \mathbf{E}[-\log P(S)] = -\sum_{S \in \mathcal{S}} P(S) \log P(S),$$

where the summation is over all distinct structures.

In order to compute the probability of a given structure S, one needs to estimate the number of ways, $N(S)$, to construct a given structure S (i.e., unlabeled graph). For this, the automorphisms of a graph is to be considered. An *automorphism* of a graph G is an *adjacency preserving permutation* of the vertices of G. The collection $\mathrm{Aut}(G)$ of all automorphisms of G is called *the automorphism group* of G. In the sequel, $\mathrm{Aut}(S)$ of a structure S denotes $\mathrm{Aut}(G)$ for some labeled graph G such that $G \cong S$. In group theory, it is well known that

$$N(S) = \frac{n!}{|\mathrm{Aut}(S)|}$$

and therefore, $1 \le |\mathrm{Aut}(S)| \le n!$.

This trivial observation leads to a relation between the graph entropy and the structural entropy [10].

Lemma 4. *If all isomorphic graphs have the same probability, then*

$$H_S = H_{\mathcal{G}} - \log n! + \sum_{S \in \mathcal{S}} P(S) \log |\mathrm{Aut}(S)|$$

for any random graph $\mathcal{G}$ and its corresponding random structure $\mathcal{S}$, where $\mathrm{Aut}(S)$ is the automorphism group of S.

In order to further advance our theory, we need to adopt a graph generation model. From now on, we assume a memoryless Erdős-Rényi model $\mathcal{G}(n, p)$ over n vertices in which edges are added independently and randomly with probability p. Thus $P(G) = p^k q^{\binom{n}{2}-k}$, where $q = 1-p$. To compute the entropy of $\mathcal{S}(n, p)$ we need to estimate $N(S)$. For this, we must study an important property of $\mathcal{G}(n, p)$, namely *asymmetry*. A graph is said to be *asymmetric* if its automorphism group does not contain any permutation other than the identity (i.e., $|\mathrm{Aut}(G)| = 1$); otherwise it is called *symmetric*. It is known that almost every graph from $\mathcal{G}(n, p)$ is asymmetric [7, 43]. In the sequel, we write $a_n \ll b_n$ to mean $a_n = o(b_n)$ when $n \to \infty$.

Lemma 5 (Kim, Sudakov, and Vu, 2002). *For all p satisfying $((\ln n)/n) \ll p$ and $1 - p \gg ((\ln n)/n)$, a random graph $G \in \mathcal{G}(n, p)$ is symmetric with probability $O(n^{-w})$ for any positive constant w.*

Using this property, we can now present the structural entropy and establish the asymptotic equipartition property (AEP), that is, the typical probability of a structure S. In [10] we prove.

Theorem 5 (Choi and Szpankowski, 2009). *For large n and all p satisfying $((\ln n)/n) \ll p$ and $1 - p \gg ((\ln n)/n)$, the following holds:*

(i) *The structural entropy H_S of $\mathcal{G}(n, p)$ is*

$$H_S = \binom{n}{2} h(p) - \log n! + O\left(\frac{\log n}{n^{\alpha}}\right), \quad \textit{for some } \alpha > 0,$$

(ii) *(AEP) For a structure $S \in \mathcal{S}(n, p)$ and $\varepsilon > 0$,*

$$P\left(\left| -\frac{1}{\binom{n}{2}} \log P(S) - h(p) + \frac{\log n!}{\binom{n}{2}} \right| < \varepsilon \right) > 1 - 2\varepsilon, \tag{35}$$

where $h(p) = -p \log p - (1-p) \log(1-p)$ is the entropy rate of a binary memoryless source.

By Shannon's source coding theorem, the structural entropy computed in Theorem 5 is a fundamental lower bound for the lossless compression of structures from $\mathcal{S}(n, p)$. However, the challenge is to design an asymptotically optimal compression algorithm matching the first two leading terms $\binom{n}{2} h(p) - n \log n$ of the structural entropy with high probability. We discuss it next.

Our algorithm, called Szip (Structural zip), is a compression scheme for unlabeled graphs. In other words, given a labeled graph G, it compresses G into a codeword, from which one can construct a graph S that is isomorphic to G. The algorithm consists of two stages. First it encodes G into two binary sequences and then compresses them using an arithmetic encoder.

The main idea behind our algorithm is quite simple (see [10] for details and Figure 7): We select a vertex of a graph, say v_1($v_1 = i$ in Figure 7), and store the *number* of neighbors of v_1 in a binary string B_1 (0100 in Figure 7). We remove this vertex, and then partition the remaining $n-1$ vertices into two sets: the neighbors of v_1 (d, f, g, i in Figure 7) and the non-neighbors of v_1 (a, b, c, e, h in Figure 7). We continue by selecting (and removing) a vertex, say v_2 ($v_2 = f$ in Figure 7), from the neighbors of v_1 and store two *numbers* in either string B_1 or B_2 (if there is only one neighbor or none): the number of neighbors of v_2 among each of the above two sets. Then we partition the remaining $n-2$ vertices into four sets: the neighbors of both v_1 and v_2, the neighbors of v_1 that are non-neighbors of v_2, the non-neighbors of v_1 that are neighbors of v_2, and the non-neighbors of both v_1 and v_2. This procedure continues until all vertices are processed. This process of selecting and splitting vertices can be described by a tree as illustrated in Figure 7.

During the construction the number of neighbors of the selected vertex is appended to either sequence B_1 or sequence B_2, where B_2 contains those numbers for singleton sets (i.e., we store either "0" when there is no neighbor or "1" otherwise). The sequence B_2 is represented by a "square" in the associated tree in Figure 7. We then compress B_1 and B_2 using an arithmetic encoder.

In [10] we prove that the algorithm just presented achieves the structural entropy *up to the first two leading terms* by showing that the length of B_2 (in compressed form) dominates the compression rate. In fact, we also observe that by the construction B_2 can be viewed as generated by a memoryless source with probability p. We prove the following.

Theorem 6 (Choi and Szpankowski, 2009). *Let $L(S)$ be the length of the codeword generated by our algorithm for Erdős-Rényi graphs $G \in \mathcal{G}(n, p)$ isomorphic to a structure S. Then: (i) For large n,*

$$\mathbf{E}[L(S)] \leq \binom{n}{2} h(p) - n \log n + (c + \Phi(\log n))n + o(n),$$

where c is an explicitly computable constant, and $\Phi(\log n)$ is a fluctuating function with a small amplitude independent of n.

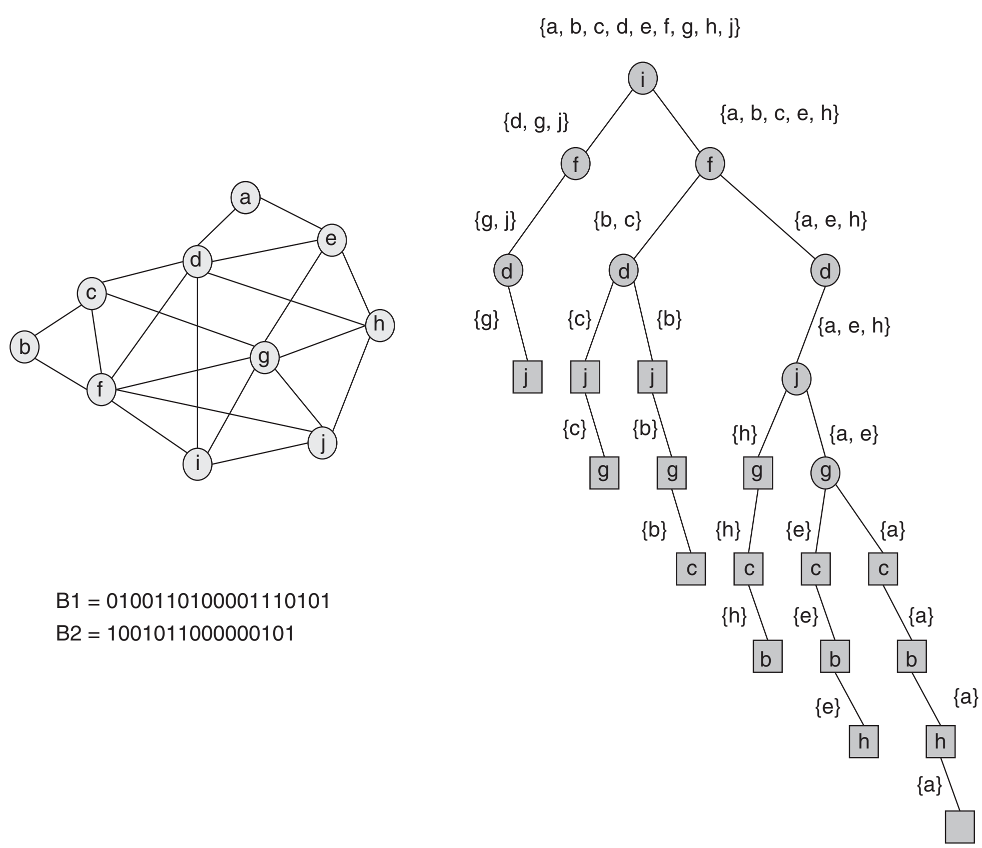

Fig. 7. Illustration to Szip.

(ii) *Furthermore, for any* $\varepsilon > 0$,

$$P(L(S) - \mathbf{E}[L(S)] \le \varepsilon n \log n) \ge 1 - o(1).$$

(iii) Our algorithm SZIP *runs either in time* $O(n^2)$ *in the worst case for any graph or in time* $O(n+e)$ *on average for graphs generated by* $\mathcal{G}(n,p)$, *where e is the average number of edges.*

In the remaining part of this section, we present a sketch of the proof of Theorem 6 (i). We need to compute the average lengths $L(B_1)$ and $L(B_2)$ of strings B_1 and B_2, respectively. These lengths can be evaluated through the associated tree T_n shown in Figure 7. In fact,

$$L(B_1) = \sum_{x \in T_n \text{ and } N_x > 1} \lceil \log(N_x + 1) \rceil \tag{36}$$

$$L(B_2) = \sum_{x \in T_n \text{ and } N_x = 1} \lceil \log(N_x + 1) \rceil = \sum_{x \in T_n \text{ and } N_x = 1} 1 \tag{37}$$

where N_x is the degree of a node x in the associated tree T_n.

To analyze $L(B_1)$ and $L(B_2)$ it is convenient to introduce an auxiliary tree that we call (n,d)-tries[3] and denote as $T_{n,d}$. The root of such a tree contains n balls (vertices of the underlying graph) that are consequently distributed between two subtrees according to a simple rule: In each step, all balls independently move down to the left subtree (say with probability p) or the right subtree (with probability $1-p$), and a new node is created as long as there is at least one ball in that node. Finally, a non-negative integer d is given so that at level d or greater one ball is removed from the leftmost node before the balls move down to the next level (in our case we set $d = 0$). These steps are repeated until all balls are removed (i.e., after $n + d$ steps). Of interest are such tree parameters as the depth, path length (sum of all depths), size, and so forth.

We compute now the averages of $L(B_1)$ and $L(B_2)$ for a randomly generated Erdős-Rényi graph. For $L(B_1)$, in the tree $T_{n,d}$ define

$$A_{n,d} = \sum_{x \in T_{n,d} \text{ and } N_x > 1} \lceil \log(N_x + 1) \rceil,$$

and then $\mathbf{E}[L(B_1)] = a_{n,0}$. Also let $a_{n,d} = \mathbf{E}[A_{n,d}]$. Clearly, $a_{0,d} = a_{1,d} = 0$ and $a_{2,0} = 0$. For $n \ge 2$ and $d = 0$, we observe that

$$a_{n+1,0} = \lceil \log(n+1) \rceil + \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} (a_{k,0} + a_{n-k,k}), \tag{38}$$

$$a_{n,d} = \lceil \log(n+1) \rceil + \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} (a_{k,d-1} + a_{n-k,k+d-1}). \tag{39}$$

To estimate $L(B_2)$ we observe that

$$L(B_2) = \sum_{x \in T_{n,0}} N_x - B_{n,0} = \frac{n(n-1)}{2} - B_{n,0}. \tag{40}$$

where $B_{n,d} = \sum_{x \in T_{n,d}, N_x > 1} N_x$. The last equality follows from the fact that the sum of N_x's for all x at level ℓ in $T_{n,0}$ is equal to $n - 1 - \ell$. Let $b_{n,d} = \mathbf{E}[B_{n,d}]$. Clearly, $b_{0,d} = b_{1,d} = 0$ and $b_{2,0} = 0$. For $n \ge 2$, we observe that to $a_{n,d}$:

$$b_{n+1,0} = n + \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} [b_{k,0} + b_{n-k,k}], \quad \text{for } n \ge 2, \tag{41}$$

and

$$b_{n,d} = n + \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} [b_{k,d-1} + b_{n-k,k+d-1}], \quad \text{for } n \ge 2, d \ge 1. \tag{42}$$

Indeed, recurrence (41) follows from the fact that starting with $n + 1$ balls in the root node, and removing one ball, we are left with n balls passing through the root node. The root contributes n since each time a ball moves down it adds 1 to the path length. Those n balls move down to the left or the right subtrees. Let us assume k balls move down to the left subtree (the other $n - k$ balls must move down to the right subtree); this occurs with probability $\binom{n}{k} p^k q^{n-k}$. At level one, one ball is removed from those k balls in the root of the left subtree. This contributes $b_{k,0}$. There will be no removal from $n - k$ balls in the right subtree until all k balls in the left subtree are removed. This contributes $b_{n-k,k}$. Similarly, for $d > 0$ we arrive at recurrence (42).

We are then faced with the reduced problem to find asymptotic solutions of two-dimensional recurrences (38)–(39) and (41)–(42). We concentrate on the latter and follow [11].

If we let $d \to \infty$ in (42) and assume that $b_{n,d}$ tends to a limit $b_{n,\infty}$, then (42) becomes

$$b_{n,\infty} = n + \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} [b_{k,\infty} + b_{n-k,\infty}] \tag{43}$$

with $b_{0,\infty} = b_{1,\infty} = 0$. This is the same as the recurrence for the mean path length in a standard *trie*, discussed above. For example, in [44, 76] it is proved that

$$b_{n,\infty} = \sum_{\ell=2}^{n} (-1)^{\ell} \binom{n}{\ell} \frac{\ell}{1 - p^{\ell} - q^{\ell}}. \tag{44}$$

The asymptotic expansion of (43) and the above as $n \to \infty$ may be obtained by a combination of singularity analysis and depoissonization arguments (see [24, 36, 76]). We obtain

$$b_{n,\infty} = \frac{1}{h} n \log n + \frac{1}{h}\left[\gamma + \frac{h_2}{2h} + \Phi(\log_p n)\right] n + o(n), \tag{45}$$

where $h := h(p)$ is the entropy, $h_2 = p \log^2 p + q \log^2 q$, γ is the Euler constant, and $\Phi(x)$ is the periodic function

$$\Phi(x) = \sum_{k=-\infty, k \ne 0}^{\infty} \Gamma\left(-\frac{2k\pi i r}{\log p}\right) e^{2k\pi r i x}, \tag{46}$$

provided that $\log p / \log q = r/s$ is rational, with r and s being integers with $\gcd(r,s) = 1$. If $\log p / \log q$ is irrational, then the term with Φ is absent from the $O(n)$ term of (45).

Let now $\tilde{b}_{n,d} = b_{n,d} - b_{n,\infty}$ measures how the path lengths in the (n,d)-trie differs from those in a regular trie. From (42) and (43), we then obtain

$$\tilde{b}_{n,d} = \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} [\tilde{b}(k, d-1) + \tilde{b}(n-k, k+d-1)], \quad \text{for } n \ge 2, d \ge 1, \tag{47}$$

which unlike (42) is a homogeneous recurrence. It turns out that the second term under the sum is negligible, which even further simplifies the recurrence. Then analytic techniques such as Mellin transform and depoissonization can be applied leading to asymptotic solution of (47).

We summarize our main result proved in [11].

[3] A trie [44, 76] is an ordered tree data structure that stores keys usually represented by strings. Tries were introduced by de la Briandais (1959) and Fredkin (1960) who also introduced the name *trie* derived from "re*trie*val".

Theorem 7. *For $n \to \infty$ and $d = O(1)$ we have $\tilde{b}(n, d) = O(\log^2 n)$. More precisely*

$$\tilde{b}_{n,d} = \frac{1}{2h \log p}\log^2 n + \frac{d}{h}\log n + \Big[-\frac{1}{2h} + \frac{1}{h\log p}\Big(\gamma + 1 + \frac{h_2}{2h} + \Psi(\log_p n)\Big)\Big]\log n + O(1), \tag{48}$$

where $\Psi(\cdot)$ is the periodic function

$$\Psi(x) = \sum_{k=-\infty, k\neq 0}^{\infty}\Big[1 + \frac{2k\pi ir}{\log p}\Big]\Gamma\Big(-\frac{2k\pi ir}{\log p}\Big)e^{2k\pi irx} \tag{49}$$

and $\log p/\log q = r/t$ is rational, as in (46). If $\log p/\log q$ is irrational, the term involving Ψ in (48) is absent. Thus

$$b_{n,0} = \frac{1}{h}n\log n + \frac{1}{h}\Big[\gamma + \frac{h_2}{2h} + \Phi(\log_p n)\Big]n + O(\log^2 n)$$

for large n.

To complete the proof of Theorem 6 we need to evaluate the $a_{n,0} = \mathbf{E}[L(B_1)]$ that satisfies the set of recurrences (38)–(39). Using the same approach as above we prove in [10, 11]

$$\mathbf{E}[L(B_1)] = \frac{n}{h}A_*(-1) + o(n), \quad A_*(-1) = \sum_{\ell=2}^{\infty}\frac{\lceil \log(\ell+1)\rceil}{\ell(\ell-1)}$$

if $\log p/\log q$ is irrational. If $\log p/\log q = r/s$ is rational, the constant $A_*(-1)$ must be replaced by the oscillatory function

$$\sum_{k=-\infty, k\neq 0}^{\infty} A_*\Big(-1 + \frac{2k\pi ir}{\log p}\Big)e^{2k\pi ir\log_p n} \tag{50}$$

where

$$A_*(s) = \sum_{n\geq 2}\frac{\lceil \log(n+1)\rceil}{n!}\Gamma(n+s).$$

Summing up, we compute $\mathbf{E}[L(S)] = \mathbf{E}[L(\hat{B}_1) + L(\hat{B}_2)] + O(\log n)$, where $\hat{B}_1$ and $\hat{B}_2$ are strings B_1 and B_2 compressed by the arithmetic encoder, while $O(\log n)$ bits are needed to encode n. The arithmetic encoder can compress a binary sequence of length m on average up to $mh + \frac{1}{2}\log m + O(1) = mh + O(\log m)$, where h is the entropy rate of the binary source. For string B_2 we know that $h = h(p)$, and this completes the part (i) of Theorem 6(i).

3.2. Acknowledgment

This work is dedicated to Philippe Flajolet who passed away in March 2011. Philippe was a friend, a colleague, and a mentor who taught many of us analytic combinatorics. I am grateful to my other "French connection", Philippe Jacquet, a friend of the last thirty something years with whom I wrote many long papers and whose cheerful attitude towards life makes research fun. I have benefited a lot from my "Palo Alto" connections: Tom Cover, whose untimely death occurred during the preparation of this paper, was a kind and great host during my sabbatical at Stanford in 1999. I thank Gadiel Seroussi and Marcelo Weinberger for many hours of discussion in HPL. Yiannis Kontoyiannis and Mark Ward kindly agreed to read this paper and complained so a better, gentle and more readable paper is presented to the readers. I am grateful to Sergio Verdu for many comments regarding my work, but mostly for his friendship and setting high standards for all of us. Finally, I thank all my co-authors for patience.

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A Report on the State of the Transactions

The IT Transactions is an extraordinarily successful journal that is in constant peril of being destroyed by its success. By any measure, it is one of the top journals in the IEEE, and indeed in applied mathematics, computer science, and engineering. Financially, it is the main revenue contributor to the IT Society.

Perhaps in part because of its success and prestige, the IT Transactions has continued to grow at a rate much greater than that of any other long-established journal of which we are aware. Six years ago a committee headed by Alexander Vardy estimated that "In ten years from now the Transactions will be publishing over 10,000 pages per year". In 2011, we published over 8000 pages and we seem to be on track to publish even more in 2012. We receive about 1100 submissions per year and our submission-to-publication time is hovering around an average of 21 months.The "thud factor" of the printed monthly volume has become embarrassing.

All this has put extraordinary pressure on Associate Editors (AEs) and reviewers. In response, we have taken the following first measures:

1) We have instituted a "fast-reject" system that now returns about 30% of submitted papers based on assessments by the editorial board. This is in line with the International Mathematical Union's Best Practices stating, among others, that: "Manuscripts that are deemed not to adhere to the journal's standards or scope can be quickly returned to the authors with a brief editorial justification."

2) We have raised standards in all editorial areas with regard not only to quality, but also to significance and relevance of papers. As a result, our acceptance rate has fallen to 40%.

3) A number of senior colleagues have kindly agreed to serve as an AE for a second time.

4) We have transitioned to the Scholar One paper handling system, which allows much closer tracking of paper status. Deadlines have been imposed for reviews and revisions. AEs and reviewers receive automated reminders. A part-time editorial assistant (Alison Larkin) has been hired, and she and I follow up with personal reminders on overdue papers as well.

5) Accepted papers now appear on IEEE Xplore within about two weeks. This "preprint," although not yet edited, has a DOI number and is fully citable.

To date, the impact of these measures remains unclear. Further measures are being discussed in the Executive Editorial Board and among the society officers.

I cannot emphasize too strongly that every member of our community who takes pride in and benefits from the success of our Transactions must pull his or her weight by agreeing to review a number of papers commensurate with the number of papers that he or she submits, and providing reviews that are not only thoughtful and professional, but also timely. As the International Mathematical Union has said in its "Best Practices for Journals:"

"Researchers who benefit from the literature and contribute to it as authors also have an obligation to participate in the peer review process, in particular by serving as referees in their areas of expertise...While no one has an obligation to review any particular paper, the decision to do so or not should be communicated in a timely fashion. Once a referee has agreed to serve, that referee should adhere to the agreed-upon schedule (including revisions) and inform the editor of unanticipated delays."

The community as a whole must help to keep our prized Transactions from foundering under its extraordinary growth, so that it can have a sustainable future.

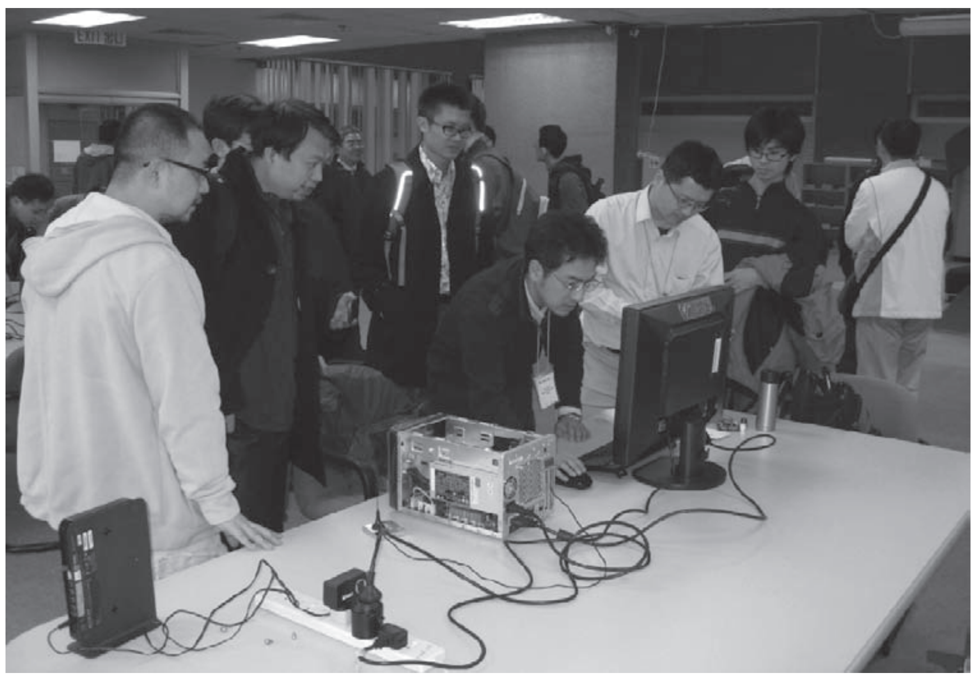
Finally, I would like to offer my most sincere gratitude to all of the Associate Editors, the Publication Editors, and the Executive Editorial Board for their hard work, dedication, and collegiality.

Yours sincerely,
Helmut Bölcskei
Editor-in-Chief
IEEE Transactions on Information Theory

SRIF 2012 Report

Soung Liew

The First Software Radio Implementation Forum (SRIF 2012; http://srif2012.inc.cuhk.edu.hk/) was held by the Institute of Network Coding (INC) of The Chinese University of Hong Kong (CUHK) on Jan 12–13, 2012.



SRIF aims to bring together researchers interested in all aspects of software radio implementations of wireless systems for exchange and sharing of ideas. Academic research efforts today in wireless communications and networking are mostly theoretical in nature. With software radio, it is now possible to prototype many of the proposed systems. These prototype efforts serve two purposes: 1) demonstration of feasibility; and 2) identification of new critical problems that demand further attention.

The barriers to the implementation efforts, however, can be substantial. The purpose of this forum is to bring together researchers and developers so that they can share their knowledge in software radio to expedite their prototyping efforts.

Toward that end, SRIF has drawn active participation from many organizations, particularly those in the Greater China region. Responses were very positive, with more than 120 registrations. Four invited talks by leaders in software radio implementation, in particular, drew wide attendance: (i) Dr. Erran Li (Bell Lab); (ii) Dr. Tom Rondeau (Gnu Radio); (iii) Dr. Kun Tan (Microsoft Research); (iv) Dr. Yang Yang (WiCo, Chinese Academy of Science).

Besides the four invited talks, there were 11 other technical talks and four demonstrations by eminent researchers in the field. One of the talks and demonstrations were on the first implementation of a Physical-layer Network Coding system by a team of researchers led by Prof. Soung Liew of INC, CUHK.

At the conclusion of SRIF 2012, the speakers and many organizing team members expressed that they gained much from the participation of the event, and expressed their interest to continue organizing this event as a regular annual event.

Slides and videos of SRIF 2012 talks can be found in http://srif2012.inc.cuhk.edu.hk/schedule.shtml.

Recent Activities of the IEEE IT Student Committee

Mustafa El Halabi, Salim El Rouayheb and Elza Erkip

The IT Student Committee is off to a good start this year. As of February 2012, the faculty chairs of the committee are Elza Erkip and Sriram Vishwanath. We would like to extend a big thank you for Aylin Yener for getting the IT Student Committee off the ground and for being an awesome chair during the past few years.

The Student Committee hosted a well–attended Roundtable Research Discussion lunch event on the second day of the CISS conference at Princeton University. More than 100 PhD students and postdocs had the opportunity to gather around nine tables to discuss their research interests and share lunch with other students. The event started with a welcome speech by Elza Erkip followed by a drawing for surprise prizes. The prizes consisted of five Barnes & Noble gift cards and the following three books that were generously donated by Cambridge Press:

1) Network Information Theory (El Gamal & Kim)

2) Information Theory (Csiszar & Korner)

3) Physical-Layer Security (Bloch & Barros)

In addition, we had a special prize this year: two lucky students had their pictures taken with our "Information Theory Celebrity" Sergio Verdu and later on received an autographed copy. Thanks to Sergio for his generosity, and congratulations to winners Jing Huang (Notre Dame) and Arun Subramanian (Syracuse). We would also like to take this opportunity to thank the following students and postdocs for leading the roundtable discussions:

- Sreechakra Goparaju (Princeton): Coding Theory
- Inaki Esnaola (Princeton): Compressed Sensing
- Pulkit Grover (Stanford): Control, Computation and Communication
- Mustafa El Halabi (Texas A&M): Cryptography and Information Theoretic Security
- Victoria Kostina (Princeton): Finite Blocklength Information Theory
- Viveck Cadambe (MIT): Interference Alignment
- Achaleshwar Sahai (Rice): Interference and Cooperation in Networks
- Salim El Rouayheb (Princeton): Network Coding
- Ravi Tandon (Princeton): Network Information Theory

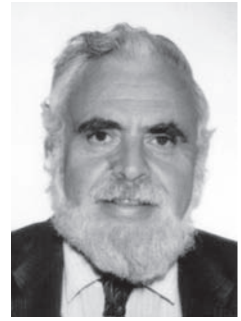
We are currently preparing for the upcoming ISIT this summer. We plan to have two lunchtime events for students, one of which will be the ISIT 2012 Student Show. This will be a new, fun event for students, fully run by students. Please contact us at sriram@ece.utexas.edu if you are interested in participating or hosting the student show. We look forward to seeing all students and postdocs in our ISIT events.

Finally, we would like to call volunteers to action. The student committee events are organized with the help of the student and postdoc volunteers; this is your committee and we need your help! Please consider being involved in the committee as a volunteer or as a student/postdoc leader. Contact us by e-mail elza@poly.edu or sriram@ece.utexas.edu if you are interested.

GOLOMB'S PUZZLE COLUMN™

Knight's Tours

Solomon W. Golomb



A KNIGHT'S TOUR on an $a \times b$ rectangular board is a sequence of moves by a "chess knight" that starts on one square of the board and lands on each of the other squares exactly once. If, from the last square visited, the knight could return, in one "knight's move", to the first square, it is called a "closed tour." Otherwise it is an "open tour."

A. The smallest $a \times b$ board that has any knight's tour is a 3×4 board.

1) Find an open knight's tour on the 3×4 rectangular board. (Only open tours exist.)

2) Prove that such a tour must begin on one corner of the 3×4 board and end on the opposite corner.

B. The 4×5 board also has (only open) knight's tours.

3) Find four different knight's tours on the 4×5 board. (Since a tour from square x to square y can be reversed to become a tour from square y to square x, two open tours that have both their end-points the same (in either order) will not be considered "different."

4) Prove that a *closed* knight's tour on *any* $4 \times n$ board is not possible.

5) Find any knight's tour (open or closed) on a 6×6 board.

6) The original "knight's tour" problem was to find a closed knight's tour on the 8×8 chessboard. Euler is widely believed to have found the first solution. Can you find one?

GOLOMB'S PUZZLE COLUMN™

Powers with Shared Digits Solutions

Solomon W. Golomb

1) The number of *powers* from 1 to 10^6 is 1111, as follows. There are $10^3 = 1000$ squares, 90 *new* cubes (since 10 of the $10^2 = 100$ cubes were already counted as squares); 11 new fifth powers (since, of the 15 fifth powers, 3 were already squares, 2 were already cubes, but 1– in both senses was already both); 5 new 7*th* powers, 2 new 11*th* powers; and one each of new 13*th*, 17*th* and 19*th* powers. Note that it was not necessary to consider *nth* powers for composite values of n.

2) The number of *non-powers* from 1 to x, by inclusion/exclusion, is given by

$$\sum_{d|Q} \mu(d) \lfloor x^{1/d} \rfloor,$$

where $\mu(\cdot)$ is the Möbius function, and summation is over all positive integer division of $Q =$ the product of all primes $\le \log_2 x$. Hence, the number of *powers* from 1 to x is

$$\lfloor x \rfloor - \sum_{d|Q} \mu(d) \lfloor x^{1/d} \rfloor = - \sum_{1<d|Q} \mu(d) \lfloor x^{1/d} \rfloor.$$

3) Here are the same-digits 3-digit powers.

a. $169 = 13^2, 196 = 14^2, 961 = 31^2$.

b. $125 = 5^3, 512 = 8^3$.

c. $144 = 12^2, 441 = 21^2$.

d. $324 = 18^2, 243 = 7^3$.

e. $256 = 4^4 (= 2^8), 625 = 5^4$.

4) Here are the sets of 4-digit powers that share the same digits.

a. $4216 = 96^2, 9261 = 21^3, 1296 = 6^4$.

b. $1764 = 42^2, 4761 = 69^2$.

c. $1089 = 33^2, 9801 = 99^2$,
$1024 = 32^2 (= 2^{10}), 2401 = 49^2 (= 7^4)$.

5) $16,384 = 128^2 (= 2^{14})$.

$38,416 = 196^2 (= 14^4)$.

$31,684 = 178^2$.

$36,481 = 191^2$.

$43,681 = 209^2$.

6) $a = 1024 = 32^2 = 2^{10}$. $a^2 = 1,048,576 = 2^{20}$.

$b = 2401 = 49^2 = 7^4$. $b^2 = 5,764,801 = 7^8$.

IT Society Mentoring Network: Call for Participation

We have identified mentoring as being an important component of success and a particular challenge for some of our society members. We will therefore continue our successful mentorship program, *open to all*. A typical mentor/mentee pairing would be a faculty member or professional in industry mentoring a graduate student or postdoc, or a senior faculty or industry researcher mentoring a junior faculty or researcher. We would strongly encourage mentees to become also mentors currently or in the future.

A mentor/mentee relationship will be a priori a two year one. A mentor will agree to communicating with his/her mentee roughly a few times per year to provide professional advice and feedback, *e.g.*, by helping the mentee with proposal writing or by introducing him to potential collaborators. The mentor/mentee list will be posted on our society web site. We are also having a yearly mentor/mentee social event at ISIT, the next one will take place at ISIT 2012 in Cambridge, MA. The only requirement for our mentoring program is that a mentor/mentee should be part of the IEEE IT Society for the duration of the mentoring period.

Anyone who is interested in joining the mentoring program (as a mentor/mentee or both) is invited to sign up at the following address: http://www.itsoc.org/people/committees/outreach/mentoring/mentoring-program

Society Outreach Committee
(Joerg Kliewer, Elza Erkip, Daniela Tuninetti, Bobak Nazer)

ICC 2013 Call for Workshops

In 2013 the IEEE International Conference on Communications (ICC 2013) will be held for the first time in Eastern Europe, in the beautiful city of Budapest. As with previous editions of this IEEE ComSoc flagship conference, ICC 2013 will be hosting a set of workshops and therefore invites submission of workshop proposals.

Workshops should emphasize current topics of particular interest, and should include a mix of regular papers, invited presentations, and panels, while in general promoting the participation of attendees in active discussion.

Website: http://www.ieee-icc.org/2013/workshops.html

Workshop proposal submission deadline: 25 June 2012

ICC Workshop Chairs:

Thomas Michael Bohnert, Technical Director, SAP Research, Zurich, Switzerland

Christoph Mecklenbrauker, Professor, Vienna Univ. of Technology, Austria

Christina Fragouli, Assistant Professor, EPFL Lausanne Switzerland

ISITA2012 HAWAII

2012 International Symposium on Information Theory and its Applications

Hawaii Convention Center, Honolulu, Hawaii, USA
October 28-31, 2012

The 2012 International Symposium on Information Theory and its Applications (ISITA2012) will take place in Honolulu, Hawaii, USA, during October 28-31, 2012. The symposium has been held every two years. Hawaii is the place where the first ISITA in 1990 and the sixth in 2000 were organized.

Topics of interest include, but are not limited to:

Error Control Coding	Coding Theory and Practice
Coded Modulation	Data Compression and Source Coding
Communication Systems	Optical Communications
Detection and Estimation	Mobile Communications
Spread Spectrum Systems	Pattern Recognition and Learning
Signal Processing	Speech/Image Coding
Rate-Distortion Theory	Shannon Theory
Stochastic Processes	Cryptography and Data Security
Data Networks	Applications of Information Theory
Multi-User Information Theory	Quantum Information Theory

Papers will be selected on the basis of a full paper (not exceeding 5 pages). The deadline for submission is planned to be March 28, 2012. Notification of decisions will be made by the middle of June, 2012.

Accepted papers will appear in the symposium proceedings. Detailed information on paper submission, technical program, plenary talks, social events, and registration will be posted on the symposium web site:

http://www.isita.ieice.org/2012/

Enquiries on
General matters: isita-2012@mail.ieice.org
Paper submission: isita-2012tpc@mail.ieice.org

Deadline for paper submission	March 28, 2012
Notification of paper acceptance	Middle of June, 2012
Deadline for final paper submission	July 18, 2012
Deadline for author registration	July 18, 2012

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Anders Høst-Madsen (U. Hawaii)

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Dept. of Electrical Engineering, University of Hawaii at Manoa

FIFTIETH ANNUAL ALLERTON CONFERENCE

ON COMMUNICATION, CONTROL, AND COMPUTING

October 1 – 5, 2012
Call for Papers

The Fiftieth Annual Allerton Conference on Communication, Control, and Computing will be held from Monday, October 1 through Friday, October 5, 2012, at Allerton House, the conference center of the University of Illinois. Allerton House is located twenty-six miles southwest of the Urbana-Champaign campus of the University in a wooded area on the Sangamon River. It is part of the fifteen-hundred acre Robert Allerton Park, a complex of natural and man-made beauty designated as a National natural landmark. Allerton Park has twenty miles of well-maintained trails and a living gallery of formal gardens, studded with sculptures collected from around the world.

Papers presenting original research are solicited in the areas of communication systems, communication and computer networks, detection and estimation theory, information theory, error control coding, source coding and data compression, network algorithms, control systems, robust and nonlinear control, adaptive control, optimization, dynamic games, multi-agent systems, large-scale systems, robotics and automation, manufacturing systems, discrete event systems, multivariable control, computer vision-based control, learning theory, cyber-physical systems, security and resilience in networks, VLSI architectures for communications and signal processing, and intelligent transportation systems.

Allerton Conference will be celebrating its Golden Anniversary this year. Because of this special occasion, the conference will be longer than its usual 2 ½ days format, to accommodate some special sessions and events in connection with the 50th celebration.

Information for authors: Regular papers suitable for presentation in twenty minutes are solicited. Regular papers will be published in full (subject to a maximum length of eight 8.5” x 11” pages, in two column format) in the Conference Proceedings. Only papers that are actually presented at the conference can be included in the Proceedings, which will be available after the conference on IEEE Explore.

For reviewing purposes of papers, a title and a five to ten page extended abstract, including references and sufficient detail to permit careful reviewing, are required.

Manuscripts must be submitted by **Tuesday, July 10, 2012**, following the instructions at the Conference website: http://www.csl.uiuc.edu/allerton/.

Authors will be notified of acceptance via e-mail by August 3, 2012, at which time they will also be sent detailed instructions for the preparation of their papers for the Proceedings.

Final versions of papers to be presented at the conference will need to be submitted electronically by October 5, 2012.

Conference Co-Chairs: Bruce Hajek and Tamer Başar

Email: *allerton@csl.uiuc.edu* URL: http://www.csl.uiuc.edu/allerton

COORDINATED SCIENCE LABORATORY AND THE DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

University of Illinois at Urbana-Champaign

Conference Calendar

DATE	CONFERENCE	LOCATION	WEB PAGE	DUE DATE
March 25–30, 2012	**IEEE INFOCOM 2012**	Orlando, FL, USA	http://www.ieee-infocom.org	Passed
May 6–9, 2012	**2012 IEEE 75th Vehicular Technology Conference (VTC2012-Spring)**	Yokohama, Japan	http://www.ieeevtc.org/vtc2012spring	Passed
May 14–16, 2012	**2012 IEEE Communication Theory Workshop (CTW 2012)**	Ka'anapali, Maui, HI, USA	http://www.ieee.ctw.org/	Passed
May 14–18, 2012	**10th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt 2012)**	Paderborn, Germany	http://www.wi-opt.org/	Passed
June 10–15, 2012	**IEEE International Conference on Communications (ICC 2012)**	Ottawa, Canada	http://www.ieee-icc.org/	Passed
July 1–6, 2012	**2012 IEEE International Symposium on Information Theory (ISIT 2012)**	Cambridge, MA, USA	http://isit12.org/	Passed
August 27–31, 2012	**7th International Symposium on Turbo Codes & Iterative Information Processing**	Gothenberg, Sweden	http://www.ee.kth.se/turbo-symposium-2012/	Passed
August 28–31, 2012	**9th International Symposium on Wireless Systems (ISWCS 2012)**	Paris, France	http://www.iswcs2012.org/	Passed
September 3–6, 2012	**2012 IEEE 76th Vehicular Technology Conference (VTC2012-Fall)**	Quebec City, Canada	http://www.ieeevtc.org/vtc2012fall/	Passed
September 3–7, 2012	**2012 IEEE Information Theory Workshop (ITW 2012)**	Lausanne, Switzerland	http://itw2012.epfl.ch/	Passed
October 1-5, 2012	**50th Annual Allerton Conference on Communication, Control, and Computing**	Monticello, IL, USA	http://www.csl.uiuc.edu/allerton/	July 10, 2012
October 28–31, 2012	**2012 International Symposium on Information Theory and its Applications (ISITA 2012)**	Honolulu, HI, USA	http://www.isita.ieice.org/2012	May 5, 2012
November 4–7, 2012	**Asilomar Conference on Signals, Systems, and Computers (ASILOMAR 2012)**	Pacific Grove, CA, USA	http://www.asilomarssc.org/	May 1, 2012
November 19–20, 2012	**5th International Workshop on Multiple Access Communications (MACOM 2012)**	Dublin, Ireland	http://www.macom.ws/	June 15, 2012
December 3–7, 2012	**2012 IEEE Global Communications Conference (GLOBECOM 2012)**	Anaheim, California, USA	http://www.ieee-globecom.org/	Passed

Major COMSOC conferences: http://www.comsoc.org/confs/index.html